The Electron as a Composition of Two Vacuum Polarization Confined Photons

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Abstract

In a previous paper "A Physical Electron-Positron Model"[1] an electron model was developed in a geometrical algebra (GA) construct developed by Doran et.al. [2] The model shows the mathematical structure, and the physical description required for the existence of an electron as a composition of two photons bound by the self-induced vacuum polarization gradient of the index of refraction. This paper will develop the mechanics of a vacuum polarization induced index of refraction, binding photons in orbit around a common center of momentum creating a net electric charge.

The concept of charge has heretofore not had a theoretical explanation, accept for some unknown substance associated with mass. This model offers the physical concept of charge created from QFT mechanics.

The combined particle is a boson with a fixed angular momentum of ½ $\hbar$. This angular momentum and the index of refraction discontinuity, provide the containment mechanisms that bind the photons together.

Introduction

The wave particle duality of particle dynamics is understood as physical aspects of particles that require both perspectives to predict the outcome of experimental tests. For the purposes of this paper we will subscribe to the wave nature of a photon as a prediction of the probability location, and the particle as a point particle with dynamics directed by a gradient in the speed of light induced by the nonlinear aspects of vacuum polarization. The physical photon is assumed to be very small.

The wave nature of the electron has been well developed by Schrodinger, Dirac and many others. The Lagrangian wave nature alone however is inadequate to describe well known measurable phenomena such as charge, size and mass

By appealing to the particle nature and the nonlinear effects of photon interaction, a composition particle that has charge, mass, spin, and size can be developed. The electron size has been a particularly difficult issue for QFT since there is an infinite singularity associated with the electron. This model should be useful in regard to resolving some of those issues. The individual photons in the model still have singular aspects, but not the infinities associated with the electron.

In Geometric Algebra (GA) the Dirac Matrices become the spacetime unit coordinate vectors, which indirectly changes the normal view of QM by defining some of the aspects QM as actually features of Lorentz covariant spacetime. Parity, time reversal, charge, positive & negative mass, become part of the spacetime structure, simplifying the mapping of the Dirac relativistic quantum representation into the eight dimensional, subalgebra of the GA spacetime representation. This allows a GA functional description of a photon. [1], which in turn allows a four-dimensional composite electron.

The authors previous paper [1], proposed a model of an electron formulated as the composition of two photons using the AG rotor structures for QM formulated by Doran et.al. [2]. (Fig. 1).

This presentation offers an electron model in that context having similarities of the atomic physical model, but relies on the gradients in the index of refraction produced by the nonlinear effects of vacuum polarization as the binding mechanism.

Sketches

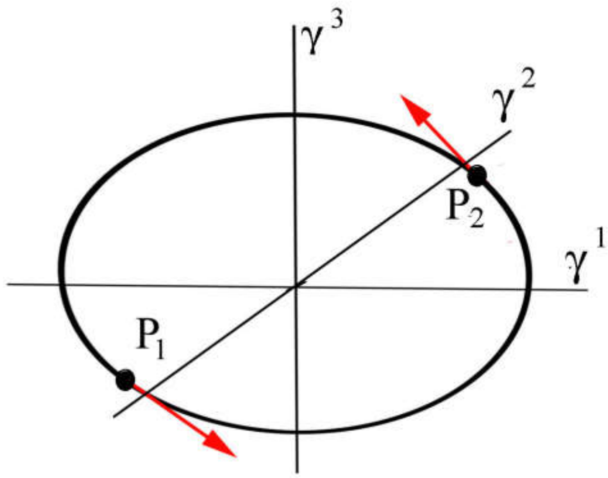

Fig.1 General configuration showing the orbiting of photons in a GA cooridnate syatem

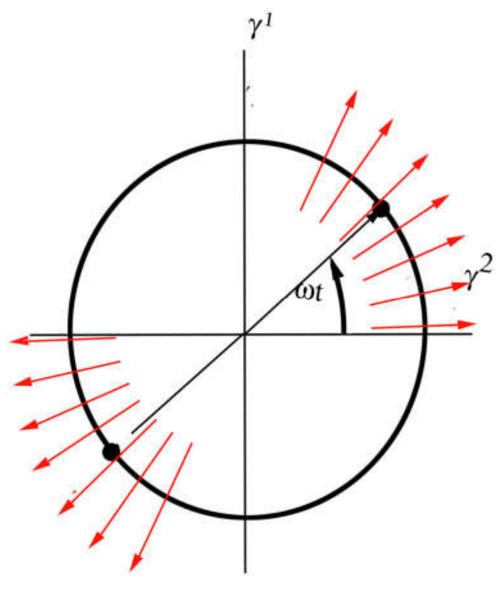

Fig.2. Radially polarized photons bound by the self-generated vacuum polarization gradient, maintaining a radially polarized circular electric field, constituting a local charge

Structure of the article

Primary Physical Mechanisms

The interaction of the two orbiting photons (Fig.1) as described in the earlier paper is not the result of an electric force, but by the motion under the influence of the mutual gradient in the index of refraction. This gradient is generated by the self-induced vacuum polarization of the two opposite going photons.

The vacuum polarization effect between two interaction photons is a well-researched process both from theoretical and experimental aspects. The first development by Sauter, Serber, Euler and others,[3],[4],[5], and later by more sophisticated methods of QFT by Schwinger and others[8],[9].
The study of the vacuum polarization on the index of refraction is quite extensive in the lower levels of E when birefringence on photons in static fields effects are predominant, [10],[11],[12],[13],[14],[15],[16],[17],[18],

[19],[20],[21]. Others have studied and proposed experiments investigated the effects of photon-photon scattering in the higher energy levels, ,[22],[23], and there have been several proposals for studies on the effects on the index of refraction by intense laser beams,[24],[25],[26],[27],[28],[29].

At the higher E levels, that are more appropriate to this work, the processes of Delbruck scattering and pair production dominates. These processes, originally proposed by Max Delbruck and first observed, by Robert Wilson [30], have been the subject of intensive research in both theoretical, and experimental since the 1950's. [31],[32], [33],[34],[35],[36], [37]. Appropriate to this paper, but not at the same energy levels is the research done by J. Kim et.al. [38], on light bending in a Coulombic field.

Vacuum polarization and the Index of Refraction

The most important aspect has been the derivation by Schwinger of the leading nonlinear corrections to the vacuum polarization that allows calculations of the local index of refraction below the critical electron–positron limit [3].

$$E_{CR} = \frac{m_e^2 c^3}{Q\hbar} = \frac{c\hbar}{Q\lambda\!\!\!^{-}{}_e^2} \tag{1}$$

At the low-energy end with non-parallel fields generally defined by the Heisenberg-Euler Lagrangian are the studies of birefringence changes in the index of refraction induced at low levels $\left(E << E_{cr}\right)$ .These have been conducted by a large number of researchers [10-21], and the results are generically similar to:

$$\eta^{\|,\perp} = 1 + \frac{\alpha\left(11 \mp 3\right)}{45\pi}\frac{E_2^2}{E_{cr}^2} \tag{2}$$

The $\|,\perp$ suffix indicates parallel and perpendicular field polarizations.

For two photons moving around a common center of momentum each experiences the electromagnetic field of the other. The relation for that interaction at $E << E_{CR}$ from Kim et.al, “Light bending in radiation background” [38], and Light bending in a Coulombic field the index of refraction can be expressed as:

$$\eta^{-1} = \frac{c}{c_0} = 1 - \frac{(14 \perp, 8 \|)\alpha^2\hbar^3}{45m^4c^5}\left(u \times E_2\right)^2 \tag{3}$$

At the higher end of the energy levels above the Kim et.al, work closer to the Schwinger limit $\left(E \sim E_{cr}\right)$, the index of refraction is better understood and by the processes related to Delbruck scattering, and pair production.

The reflection coefficient expressed in the relative index of refraction and the high end scattering experiments, lead to the conclusion that the index of refraction has an infinity at the Schwinger Limit. With multiple loops and higher order corrections the index of refraction at the higher fields as developed by Dietrich et.al. [12] is: $E \rightarrow E_{cr}$ index of refraction η^{-1} is:

$$\eta^{-1} \rightarrow \left(1 - Q\frac{B^2}{2B_{cr}^2}\sin^2\theta\right) \tag{4}$$

At very high levels of the fields the Q factor $\rightarrow 1/2$, and if E represent the maximum of the E & B fields of photons near the Schwinger limit then for two opposite colliding photons with fields of E_1 and E_2 the maximum local index of refraction is:

$$\eta^{-1} \rightarrow \left(1 - \frac{E_1 \bullet E_2}{E_{cr}^2}\right) \tag{5}$$

Index of Refraction for Photons containment

The index of refraction to maintain photons in a circular path can be determined from classical physics by variation methods applied to Fermat's principle. It is straight forward and well done by J. Evans, et.al. [39], and for stable orbits Fermat's principle requites the index of refraction to be proportional to1/r, thus in terms of the Compton radius for an electron, This can be written as:

$$\frac{c}{c_0} = \frac{kr}{\lambda\!\!\!^{-}_e} \tag{6}$$

k is the index of refraction constant and $\lambda\!\!\!^{-}_e$ is the Compton radius of the electron

Putting this into Eq.(5), the relations between c, r and E is:

$$\eta^{-1} = \left(\frac{c}{c_0}\right) = \left(\frac{kr}{\lambda\!\!\!^{-}_e}\right) = \left(1 - \frac{E^2}{E_{cr}^2}\right) \tag{7}$$

At $r = 0$, $E^2 = E_{cr}^2$, and at $E = 0$:

$$\frac{kr}{\lambda\!\!\!^{-}_e} = 1 \tag{8}$$

As $E \to 0$ the photons are no longer in contact, and $c \to c_0$ showing the index of refraction binding the photons in circular motion vanishes. The value of the index of refraction constant, k can be determined by the angular momentum of the system.

The value of c as a function of r from Eq.(7), is:

$$c = c_0 \frac{kr}{\lambda\!\!\!^{-}_e} \to r = \frac{c}{c_0}\frac{\lambda\!\!\!^{-}_e}{k} \tag{9}$$

Although the radial polarized photons will be shown to have opposing electric field, the binding mechanism of the photons is not the electric field, but the gradient in the index of refraction induced by the vacuum polarization.

Period, Frequency, & Gradient

For a photon P, orbiting in an index of refraction that is proportional to the radius of the orbit, the orbital period of revolution is constant for all radii[39].

$$\omega = \omega_P \qquad T = \frac{2\pi \lambda\!\!\!^{-}_P}{c_0} \tag{10}$$

Orbiting photons in such a configuration maintain their original free space frequency at any orbital radius, and their original frequency energy relationship:

$$\omega = \varepsilon_0 / \hbar \tag{11}$$

Spin Determined Index of Refraction Constant k

The angular momentum for the orbiting photons is properly calculated by the Path integral methods integrating the sum of the nonlinear action over all possible paths. By knowing however that the sum of the spin angular momentum of the two spin-one vector boson in a composition particle has to be $1/2\,\hbar$ the index of refraction constant k of Eq.(7), can be evaluated.

The classical angular momentum for two photons orbiting around the center of momentum perpendicular to the orbit is:

$$S = 2\frac{p}{\hbar} r \tag{12}$$

The momentum for each of the photons is $p = \varepsilon / c$ can be evaluated from the photon energy. Noting that the sum of the two photons energy and thus frequency (i.e. the Compton frequency) of the electron:

$$2p = 2\frac{\hbar\omega_p}{c} = \frac{\hbar\omega_e}{c} = \frac{\hbar c_0}{c\lambdabar_e}$$

Putting the value of r as function of c from Eq.(9), & Eq.(12), and noting that the spin for the system must be a constant ½$\hbar$ gives:

$$\frac{S}{\hbar} = \frac{1}{\hbar} p r = \frac{1}{\hbar}\frac{\hbar c_0}{c\lambdabar_e}\frac{c}{c_0}\frac{\lambdabar_e}{k} = \frac{1}{2} \tag{13}$$

The value of the index of refraction constant, k for the composite particle to have the proper spin is then:

$$k = 2 \tag{14}$$

In Eq.(12), as the value of r exceeds $r = \lambdabar_e / 2$ the value of E becomes imaginary and the angular momentum is no longer constant.

$$L \to \frac{\hbar}{\hbar\lambdabar_e} r > \frac{1}{2} \tag{15}$$

This provides the limiting binding, and confining condition for the photons, setting a maximum orbital radius to be:

$$r_{max} = \frac{\lambdabar_e}{2} \tag{16}$$

From Eq.(7), & Eq.(23), as the value of r exceeds $\lambdabar_e / 2$ the value of the interaction of E of the two photons becomes imaginary.

Figure 3, shows the relationship between the speed of light and the radius from the center of momentum.

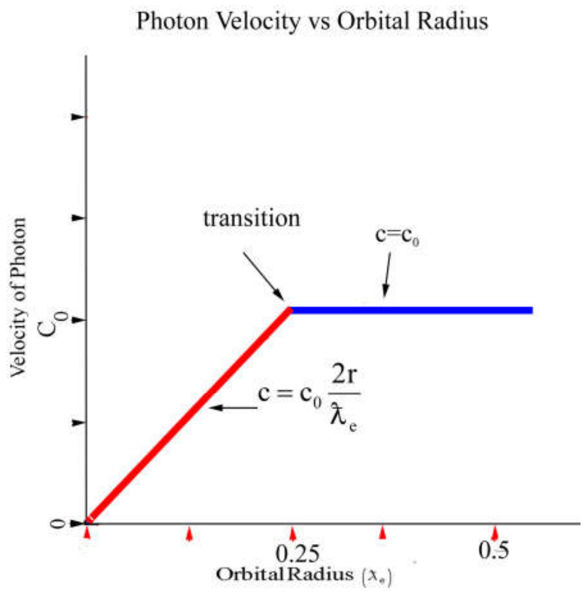

Fig. 3. This is the velocity of light experienced by the photons in orbit around the center of momentum. The value is a linear function of r below $ƛ_e / 4$ and is constant c_0 above that. At $ƛ_e / 4$ there is a discontinuity in the index of refraction, and this is the location of the boundary separating the real photons that constitutes the mass, from the Feynman probability photons that generate the electric field effects.

Vector Orientation, Stability, and Net Charge

It has been asserted in the earlier paper that the rotating photons can have electromagnetic vectors maintaining a constant radial direction along the radial vector. Two physical mechanisms dictate this: One is the Thomas precession which counteracts the photon helictical rotation, and the other is the maximizing of the vacuum polarization energy density along the radial rotation axis.

Photon Radial Alignment: Thomas Precession

Thomas Precession is a well understood phenomenon totally within the mechanics of Lorentz dynamics.
As a particle rotates around a center axis there is a frame rotation such that when it arrives back at a defined point its helictical phase orientation will

also have been rotated. For a photon in a circular index of refraction this will mean that for every cycle of rotation about the axis its helictical rotation will be reduced by that number of cycles. At half the Compton radius of the photon, this reduces the helictical frequency to zero leaving the photon with a constant radial electric vector. The half Compton radius of rotation for the photon is due to the fact that its spinor definition [1] completes after two complete rotations. This rotational radius is then the same as the Compton radius for the electron. (See **Appendix I** for details.)

E Field Maximizing and Vector Orientation

From the center of momentum frame of two identical orbiting photons, the photons move in opposite directions, and are in effect in colliding. The momentums are opposite, the velocities are opposite, and thus by CPT in the center of momentum frame, the electric field contribution to the vacuum polarization are anti aligned. For the purpose of vacuum polarization the sum of the opposite moving electric fields are additive to the field strength.

$$\mathrm{E} = \mathrm{E}_1 - \left(-\mathrm{E}_2\right) = \mathrm{E}_1 + \mathrm{E}_2 \tag{17}$$

The energy density ε, for the colliding photons is maximal for a head-on collision and expressed in 3 vector notation is[11]:

$$\varepsilon = \mathrm{E}^2 + \mathrm{B}^2 - 2\mathrm{S}\bullet\boldsymbol{k} - \left(\mathrm{E}\bullet\boldsymbol{k}\right) - \left(\mathrm{B}\bullet\boldsymbol{k}\right) \tag{18}$$

For the case of two equal & opposite photons, all but the first square terms of the energy density vanish, and in addition the birefringent terms of the Lagrangian first loop also vanish.

$$\begin{aligned} L &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\mathrm{E}^2 - \mathrm{B}^2\right) = 0 \\ L &= -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2}\left(\mathrm{E}\bullet\mathrm{B}\right) = 0 \end{aligned} \tag{19}$$

From Eq.(17), and Eq.(18), the maximal energy density for the two photons occurs when the electric and magnetic vectors are parallel, thus the electric and magnetic vectors add without any birefringent terms:

$$\varepsilon = \left(E_1 + E_2\right)^2 + \left(B_1 + B_2\right)^2, \tag{20}$$

The maximum energy density at the location of a single photon is when the square of the sums of the second photon maximizes at that point. That is the contribution to the electric density at P_1 by P_2 is when this sum maximizes. (Fig, 2)

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$$\varepsilon_1 = \left(E_1 + E_2 \sin\phi\right)^2 + \left(B_1 + B_2 \cos\phi\right)^2 \tag{21}$$

This occurs when the radial vectors and the electromagnetic vectors from P_2 are at a 45 degree angle.

$$\frac{d\varepsilon_1}{d\phi} = E^2\left(\sin\phi - \cos\phi\right) \rightarrow 0 \text{ @ } \phi = 45^0 \tag{22}$$

The result is the same from the perspective at P_2

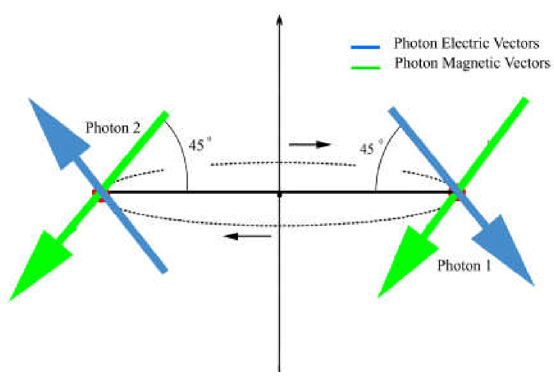

Fig. 4, Orientation of E and B vectors orbing a common center.

The combined effect of the Thomas precession and the vector orientation for the maximal field strength gives the stability to the composite electron.

As these vectors presses around the orbit there is a time average net spherical electric vector, (blue), in the 4π radial directions. The time average of the magnetic vectors, (Green) is dipolar 2π. These time integrals give the net charge to the composite particle and a magnetic dipole moment.

The model defined in electric vectors should only be considered as a classical visual description, whereas the actual electric field effects are generated by probability distribution density of the polarized photons of the Feynman action paths that escape the boundary and populate the volume outside the orbital radius.

Orbits

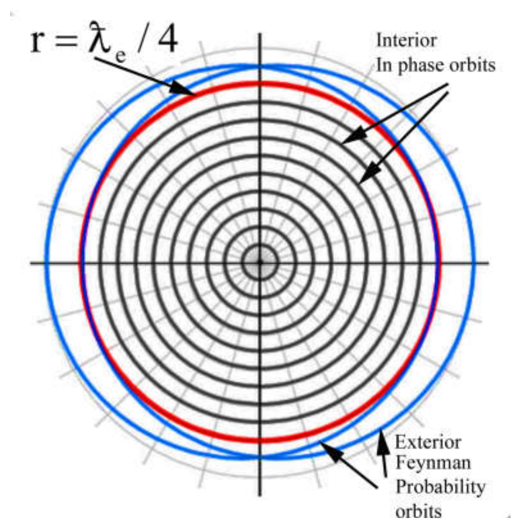

Fig.5. This sketch illustrates points on the interior radius have a circular stable orbit obeying Fermat's principle. All inside action orbits, circular, as well as elliptical are in phase and contribute to the total action. The spin angular momentum confines photon paths to radii less than 1/4 of the Compton radius. Only Feynman probability paths responsible for electric interaction exist outside this radius.

Vacuum Polarization Cross-Section

The implication of Eq.(7), is that the vacuum polarization cross-section of a photon has a sharp edge at a diameter of $D = \lambda\!\!\!^{-}_e / 4$. Graphically this cross-section interaction has a linear energy density contribution until the second photon edges separate.

$$E^2 = E_{cr}^2 \left(\frac{\lambdabar_e^2 - 4r^2}{\lambdabar_e^2} \right) \tag{23}$$

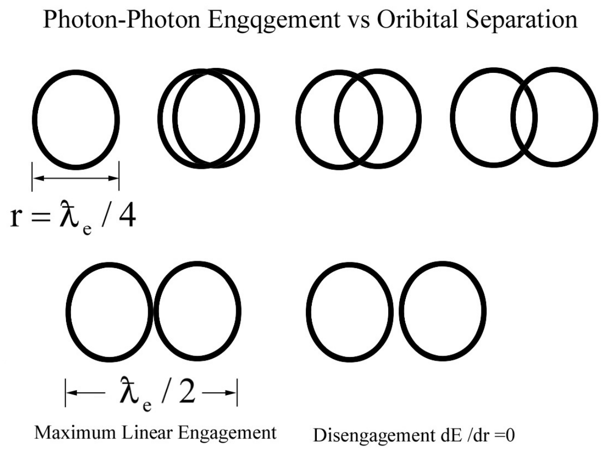

Fig.6. Cross-section engagement of orbiting photons
The photon energy density product outside1/4 of the Compton radius vanishes.

Conclusion

A model for an electron has been presented that physically demonstrates mass, charge, & spin, within the concepts of currently known physics. Nothing has been postulated that isn't well understood in terms of current physics. Neither of the physical regimes of QM or Classical physics have been stretched, compromised, or extended beyond that which has been experimentally confirmed.

It gives a physical insight to the mechanical process, and since there are no singularities associated with the model. This structure may allow QFT a path around the infamous renormalization without having to cancel infinite values with infinite values,

A physical concept of charge has been presented that addresses an ongoing dilemma for physics since inception, both in its creation and its connection to mass. This model shows charge as a photon property not at all outside the bounds of physical processes.

As is well known pair production is the result of interaction photons, this represents the mechanics of that process.

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Appendix I

Thomas Precession

As a pair of photons rotates around the center of a circle due to a variable index of refraction, the Thomas precession reduces the helictical rotation frequency of the photon. The photons frequency is reduced by exactly the axial frequency of the rotation. As the gradient in the index of refraction is increased the sum of the frequencies must remain constant.

As the circumference is reduced to the wavelength the helictical frequency is stopped. The rotation frequency is then equal to the original free particle frequency of the photon and the photon electromagnetic vectors are polarized along the orbital radius.

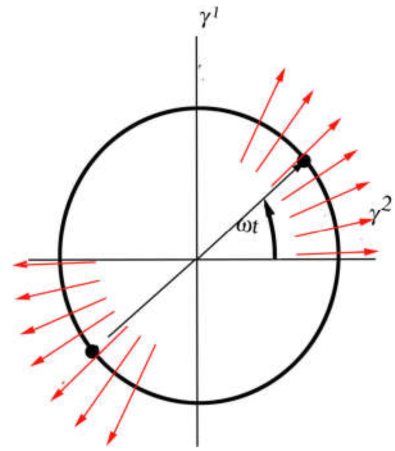

Fig.2 Two radially polarized photons bound by the self-generated vacuum polarization gradient in the index of refraction. The radially polarized circular directed electric vectors constitute the effect of a local charge.

This is easily shown from Lorentz geometric principles, the Thomas reduction to the frequency of an orbiting photon is:

$$\omega_T = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right) a \times v \tag{24}$$

a is the circular acceleration dr/dt in the moving frame thus:

$$\Delta t' = \gamma\, \Delta t \tag{25}$$

and for a photon moving in a variable index of refraction the precession is:

$$\vec{\omega}_T = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right) a \times v \rightarrow \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right)\frac{dv}{\gamma dt} \times v = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right)\frac{dv}{dt} \times v$$

For the photon the circular acceleration is:

$$\frac{dv}{dt} = \frac{c^2}{r} \tag{26}$$

And for the photon orbiting at the Compton radius:

$$r = \frac{c}{\omega_P} \tag{27}$$

Thus as the radius is reduced to the Compton radius the Thomas precession frequency reduces the helictical frequency to zero, whereas the axial frequency in the orbit plane $\vec{\omega}_R$ becomes equal to the free photon frequency.

$$\vec{\omega}_T = \omega_P \uparrow \tag{28}$$

The Thomas precession thus establishes a radial polarization along the radius vector to the center of momentum.