A Physical Electron-Positron Model in Geometric Algebra

D.T. Froedge

Formerly Auburn University
Phys-dtfroedge@glasgow-ky.com

V041917
@ http://www.arxdtf.org

Abstract

This paper is to present a physical model of the Electron & Positron particles constructed as the interaction of two photons

The photons and subsequently a model of the Electron will be defined in the math of Geometric Algebra using, and expanding on the correspondence relations between GA and QM developed by Doran, & Lasenby [3]. The vector constructs defining the electromagnetic components of a quantum system can be extended to define the physical structure of a particle. By defining a complete physical vector boson i.e. the photon, in terms of a GA vectors, is straightforward to show that an electron can be modeled as an interaction of two such photons and has the known physical attributes of an electron. The attributes include mass, spin, & charge. A clear advantage of the model is the absence of infinities that are dealt with in QFT, by the process of renormalization. The infinities of a point electron is supplanted by two point vector bosons that do not have an infinity and operate under the rules of QFT.

As in the Bohr model of the atom, the classical dynamics illustrate what are in effect the integral paths of QFT; it should not be unreasonable to expect that the dynamics of photon interaction defined by GA can illustrate electron structure. The probability aspect of QFT as it relates to the photon is not in question and still is the underlying mechanism of quantum probability.

Introduction

The electron model will be shown to be the composition of a pair of interacting vector bosons circling the center of mass, held in orbit by the interaction of their own electric vectors that in turn generates a net charge as viewed from the exterior. The constructs and terminology used here are generally the same in as standard GA, as defined by Doran, & Lasenby [3], particularly the connection to the vector Lagrangian in the action. The notation will include aspects of the Feynman Slash notation [2].

To look at this as an alternative to QM would be a mistake, since the photon action defined by the rules of QFT is the basis of the charge interaction. By having a physical model of the electron makes the interaction between charged particles more transparent.

Order of Presentation

Photon Momentum and Lorentz Invariant Proper Mass

This develops the fact that the null four-momentum of photons in GA can define the momentum of a massive particle

Action Four-Vector

This develops the action four-vector as a vector Lagrangian exponent of the particle function necessary for a GA representation. This sets the departure of QM and GA, from the scalar Path Integral formulation of QFT to vector path integrals.

The Mapping of a QM Representation into a GA Electromagnetic Vector Representation

This develops the mapping from a scalar formulation to a vector representation for particle functions via the procedures developed by Doran- Lasenby

The Photon Model

This section extends the particle function for a photon from pure unitary vector function, to a vector scalar Gaussian representation by adding the square of the action.

Dirac and Klein-Gordon Solution of the Physical Photon Model

The electron Model

This develops a physical GA model for an electron as the interaction of two photons the model illustrates the mass the spin the source of positive and negative electrical charge, the annihilation, the lack of infinities, and being a solution to the Dirac and KG equations.

The Electron Systemfunction

This proposed the Systemfunction for the Electron and Positron

Terms in the Electron Systemfunction

This illustrates the individual terms in the function including the dynamic rest mass and spin contribution to the rest mass

Pair annihilation

This illustrates that the Electron-Positron annihilation results in two opposite going opposite spin photons.

Proposed Physical Electron Model

This illustrates a model of an electron having two revolving photons around a common center of mass. The photons have opposite, in phase electric vectors pointing to the center of mass. The opposite electric vectors provide the binding force between the photons.

Electron Model properties

Electric Charge
Charge Sign
Magnitude of Binding Force
Solution to the Dirac & KG Solutions
Spin

Proposed Neutrino Model (*speculation)*

This is a bit of speculation but using the Systemfunction for the electron with a slight modification of the actions a light speed particle having neutrino properties of spin and mass is presented.

Presentation

Photon Momentum and Lorentz Invariant Proper Mass

The four momentum of a particle is the instantaneous value of the derivative of the action of a particle, and thus a starting point for physical particle properties

If the mass of a photon is set to be $m = \hbar\omega / c$, the null four vector momentum of a photon can be written in GA vectors as:

$$\not{P}_1 = m_1 \left(\gamma^k c_{1k} + \gamma^0 c \right) \tag{1}$$

The mass of the spin one photon is a mnemonic and can be substituted at will for the frequency

It has been shown by the author that opposite going photons locked together exhibit dynamical propertied s of massive particles, [4] thus a starting point for dynamical particles.

For two photons going in opposite directions in Minkowski space, the momentum of each is:

$$\not{P}_1 = m_2\left(-\gamma^k c_{1k} + \gamma^0 c\right) \tag{2}$$

$$\not{P}_2 = m_2\left(-\gamma^k c_{2k} + \gamma^0 c\right) \tag{3}$$

And the sum is:

$$\not{P} = \not{P}_1 + \not{P}_2 = (m_1 + m_2)\gamma^0 c + (m_1 - m_2)\gamma^k c_{1k} \tag{4}$$

Squaring:

$$\left(\not{P}_1 + \not{P}_2\right)^2 = \not{P}_1^{\,2} + \not{P}_2^{\,2} + \not{P}_1 \not{P}_2 + \not{P}_1 \not{P}_2 \tag{5}$$

The first two terms are null. The product of two null invariants is a Lorentz scalar and thus invariant under a Lorentz transformation. [5]. It is also the commutator of the momentum vectors.

Specifically multiplying the vectors of Eq.(1), gives:

$$\left(\not{P}_1 + \not{P}_2\right)^2 = 4m_1m_2c^2 = m_0^2c^2 \tag{6}$$

This the square of the sum of two null vectors, and defines $4m_1m_2$ as the square of an invariant mass term or rest mass associated with the opposite going photons.

Eq.(4), can also be written as:

$$\not{P} = (m_1 + m_2)\left(\gamma^0 c + \frac{(m_1 - m_2)}{(m_1 + m_2)}\gamma^k c_{1k}\right) \tag{7}$$

For two such particles the velocity of the center of gravity v_C can be determined from:

$$\left(m_1 + m_2\right) v_C = m_1 c - m_2 c \tag{8}$$

or:

$$\frac{v}{c} = \frac{\left(m_1 - m_2\right)}{\left(m_1 + m_2\right)} = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2} \tag{9}$$

The vector momentum for the two photons can then be written;

$$\not{P} = m\left(\gamma^0 c + \frac{v}{c}\gamma^k c_{1k}\right) \tag{10}$$

Squaring the four momentum for the two photons Eq.(7), and writing in terms of the velocity of the center of mass becomes the standard invariant mass form:

$$\not{P}^2 = m^2 c^2 \left(1 - \frac{v^2}{c^2}\right) = m_0^2 c^2 . \tag{11}$$

This is the best indicator that two bound sped of light particles can be considered to have a proper rest mass.

Action Four-Vector

The instantaneous four-derivative of the action must necessarily be the four vector momentum defined above Eq(3), thus:

$$\not{\partial}\not{S} = \not{P} \tag{12}$$

where:

$$dS = mc\left(\gamma^0 d\left(ct\right) + \gamma^k dx_k\right) \qquad dx_k = c_k t \tag{13}$$

This is a four vector Lorentz invariant action.

The particle action of the two photons Eq.(2), and Eq.(3), is then.

$$S_1 = m_1\left(\gamma^k c x_k + \gamma^0 cct\right) \tag{14}$$

$$S_2 = m_2\left(-\gamma^k c x_k + \gamma^0 cct\right) \tag{15}$$

And the instantaneous particle action of the two photons Eq.(7), is:

$$\not{S} = \not{S}_1 + \not{S}_2 = m\left(\frac{v}{c}\gamma^k c_1 x_k + \gamma^0 c^2 t\right) \tag{16}$$

This is a null vector action and the differential, $\not{P}$, is also null.

The Mapping of a QM Representation into a GA Electromagnetic Vector Representation

In the Path Integral formulation of QM, the amplitude of the probability for the m particle to transition from one state to another is the integral over a scalar action between the two states over all possible paths. The path integral depends on the final coordinate and time in such a way that at an end points it obeys the Schrödinger equation, [2], thus heuristically for a Schrödinger wavefunction:

$$\psi(x_f, t_f) = \int dx_i \left[\int Dx(t) e^{i\int \frac{L}{\hbar} dt}\right] \psi(x_i, t_i) = \int dx_i \left[\int Dx(t) e^{i\frac{S}{\hbar}}\right] \psi(x_i, t_i)$$

The bracketed term is the propagator for the wavefunction moving over all paths from an initial state *to* a final state. The action is scalar, the exponential terms are Euler i.e. squares are negative, and there is no spatial definition of the defined particle.

There has not been a lot of development on the direct connection between a GA representation of particle physics and QM. A notable correspondence between the two is the work done in chapter 8 of Doran, & Lasenby text.[3], and that correspondence forms the basis of the math correspondence used here.

Doran et.al. [3], have mapped the Dirac relativistic quantum representation of QM into the eight dimensional subalgebra of the GA spacetime representation, a most significant transition is the transforming the scalar action representation to a vector action and vector Lagrangian (p284).

$$e^{i\frac{S}{\hbar}} \rightarrow e^{I_3 \sigma^k p \bullet x} \tag{17}$$

In the notation for action defined earlier in Eq.(14), Eq.(15), & Eq.(16), their GA corresponding representation can be written as:

$$\psi = e^{I_3 \not{S}} \qquad \not{S} = \int \not{P} dt \tag{18}$$

For the free particle under discussion, $\not{P}$,is the vector Lagrangian, and I_3 is the grade three pseudoscalar $\gamma^1\gamma^2\gamma^3$.

This mapping produces a physical difference in the understanding of the physical phenomena. The GA view moves the probabilistic issues from a global scalar particle down to the QFT probability level of multiple paths of vector bosons.

The connection made by Doran et.al. between circularly polarized electromagnetic waves and the corresponding electron model makes it clear that the vectors described for the electron are the electromagnetic vectors. This paper will exploit the vector properties of the electromagnetic fields of the photon and the vector Lagrangian to develop an electron model.

The Equations of this paper will show the electric vectors but the magnetic vectors are left out but easily included, as shown by Doran.

The Photon Model

The function, Eq.(18), with $\not{S}$ defined in Eq.(14), can well describe known properties of a spin 1 vector boson (a photon), the electric vectors spin etc, but it has no size, and its wave values exist to infinity in all directions. QFT can define the parameters from point to point and make arguments regarding the physical size, but mechanically this is not a useful description. A logical and physically descriptive model of a photon can be defined by including the square of the action $\not{S}$ into the exponent of the vector model.

$$\bar{\Theta}_{F1} = e^{-\left(\not{S}_1^{\,2} + I_3 \not{S}_1\right)} \qquad (19)$$

This is a physical Gaussian having the square of a null vector exponent traveling at c with a size that is a function of the wavelength. For GA this is the scalar constant magnitude of the Rotor. For a free particle it is presumed that the path integral solutions from one state to the next, start and end at points defined at $n\lambda$ and is not continuous.

The time reversal or opposite going photon would then be:

$$\bar{\Theta}_{F2} = e^{-\left(\not{S}_2^{\,2} + \not{S}_2 I_3\right)} = e^{-\left(\not{S}_2 + I_3\right)\not{S}_2} \qquad (20)$$

For the purposes here Eq.(19), is defined as a “Systemfunction” $\bar{\Theta}_F$ of the photon to make a distinction between a GA and QM representation. I_3, is the grade three pseudoscalar, $\gamma^1\gamma^2\gamma^3$, and is the spin orientation vector. From the GA perspective these functions are noted to be rotors.

The action square defines a Gaussian kernel having a velocity of c with the maximum value of 1 at the center and a half width of about 1.7 wavelengths. From a QFT perspective this Gaussian kernel is presumed to be defined by the probability density of the path integral. The paths are summed over all space paths and thus the influence of the photon exceeds the dimensions of the kernel. The particle physically exists on the inside of the light cone where $ct > x$.

Dirac and Klein-Gordon Solution of the Physical Photon Model

Taking the four-derivatives of the Electron Systemfunction Eq., yields:

$$\not{\partial}\,\bar{\Theta}_F = \bar{\Theta}_F\left[\not{\partial}\,\not{S}^2 + \not{\partial} I_3 \not{S}\right] \tag{21}$$

From Eq.(12), $\not{\partial}\,\not{S} = \not{P}$. And since the photon vector momentum is a null vector:

$$\left|\not{P}\right| = 0 \tag{22}$$

For physical reasons the differential of the square of the magnitude of $\not{S}^2$ is zero:

$$\not{\partial}\,\not{S}^2 = 0 \tag{23}$$

From QFT the action is a sum of path integrals over all space, but the magnitude is only defined at singular state values at the integral end points of the path $n\lambda$, thus not continuously differentiable. The derivative of $\not{S}$ however can maintains the instantaneous sum of all the paths and is differentiable.

Thus:

$$\not{\partial}\,\bar{\Theta}_F = \bar{\Theta}_F I_3 \not{P} = 0\,, \tag{24}$$

and the second derivative is:

$$\not{\partial}\,\not{\partial}\,\bar{\Theta}_F = \bar{\Theta}_F\,\not{P}^2 = 0 \tag{25}$$

Since $\not{P}$ for the photon is a null vector, both of these expressions are zero thus the model satisfies both the Dirac and Klein Gordon expressions for a photon.

The electron Model

A very significant point in the vector representation of the electron by Doran is that a comparison with the photon function in Eq.(18), with the circular polarized electromagnetic waves shows the vectors are identical thus the assertion can be made that the rotating vector of the photon are the electromagnetic vectors of the photon. It is the interaction of the electric vectors between the photons that produces a binding force and reduces the spin one to a half spin particle.

In building a the model of the electron, knowing that at the co-location of an electron-positron pair is the same as the co-location of two opposite going photons, gives the connection to allows the Systemfunctions to be deduced.

When the particles are located at the same event the Systemfunction must be factorable into an electron positron pair or a pair of opposite going photons i.e.:

$$\bar{\Theta} = \bar{\Theta}_{F1}\bar{\Theta}_{F2} = \bar{\Theta}_{E}\bar{\Theta}_{P} \qquad (26)$$

The Systemfunctions for the photons have already been defined Eq.(19), and thus the Electron-positron can be constructed from that.

The Electron Systemfunction

Rather than to construct the Electron and Positron Systemfunction from the photons, the Systemfunctions will be proposed and then deduce that it is the same.

For the Electron the proposed Systemfunction is:

$$\bar{\Theta}_E = Ae^{-\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]/2} \tag{27}$$

And the Positron is:

$$\bar{\Theta}_P = Ae^{-\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]\left[\left(\not{S}_1 - \not{S}_2\right) - I_3\right]/2} \tag{28}$$

Notable is the second factor in the Positron is a space-time reversal of the second factor in the electron. Since the corresponding photons in the Electron and photon are equal, the designation is the same.

Explicitly the exponents are:

$$\bar{\Theta}_E = (\exp)\frac{-\left[\left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) + \left(\not{S}_1\not{S}_2 + \not{S}_2\not{S}_1\right) + 1 \;+\; I_3\left(\not{S}_1 + \not{S}_2\right) + \left(\not{S}_1 + \not{S}_2\right)I_3\right]}{2} \tag{29}$$

$$\bar{\Theta}_P = (\exp)\frac{-\left[\left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) - \left(\not{S}_1\not{S}_2 + \not{S}_2\not{S}_1\right) - 1 \;-\; \left(\not{S}_1 + \not{S}_2\right)I_3 + I_3\left(\not{S}_1 - \not{S}_2\right)\right]}{2} \tag{30}$$

Pair annihilation

Multiplying the functions and noting $\not{S}_2 I_3 = -I_3 \not{S}_2$, gives just the product of two photons previously defined:

$$\bar{\Theta}_{F1}\bar{\Theta}_{F2} = \bar{\Theta}_E\bar{\Theta}_P = \left[e^{-\left(\not{S}_1^{\,2} + I_3\not{S}_1\right)}\right]\left[e^{-\left(\not{S}_2^{\,2} + \not{S}_2 I_3\right)}\right] \tag{31}$$

The null $\not{S}_1^{\,2}$, and $\not{S}_2^{\,2}$ action vectors going in opposite directions and the times are reversed for the linear vectors, thus these functions represent opposite going photons and the proper annihilation is demonstrated for the Positron Electron pair.

Terms in the Electron Systemfunction

The terms in the electron Eq.(29), can be easily identified:

The Scalar Gaussian magnitude:

$$\rho = \exp - \frac{\left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) + \left(\not{S}_1\not{S}_2 + \not{S}_2\not{S}_1\right) + 1}{2} \tag{32}$$

The first bracket is the null vector actions that establish the Gaussian kernel for each of the photon four-vector action.

The second bracket is the commutator of the photon action. Since the product of two Lorentz null vectors is a Lorentz constant, this is a constant or twice the dot product of the opposite going particles. Using Eq.(5), this is just a non kinetic rest mass, and for spin one photons the value of this term is:

$$\frac{\left(S_1S_2 + S_2S_1\right)}{2} = \frac{2S_2 \bullet S_1}{2} = \frac{2m_1m_2c^2\left[\lambda\!\!\!^{-}_1\lambda\!\!\!^{-}_2 + \left(cT\right)^2\right]}{2} = \frac{m_0^2c^2\lambda\!\!\!^{-}_C^{2}}{2} = \frac{1}{2}$$

(33)

The square of the pseudoscalar I_3 represents the product of the Spin-Spin interaction between the photons and is a spin angular momentum contribution to the rest mass.

The scalar Gaussian can then be written as:

$$\rho = \exp - \left(\frac{\left(\not{S}_1^{\,2} + 1\right) + \left(\not{S}_2^{\,2} + 1\right)}{2}\right) \tag{34}$$

To illustrate the physical properties, time can be set to zero for the actions in Eq.(34), then the location of the Gaussian maximum for each of the photons can be viewed as being is one wavelength, or one complete revolution, ahead of its null position, or exactly in coincidence with its null value.

$$-x_1^2 + 1 = 0 \qquad -x_2^2 + 1 = 0 \tag{35}$$

In the case of the Positron Eq.(30), the peak of the Gaussian peak is behind the null location by one revolution.

The Vector Terms:

The two defined rotors for the particles in Eq.(29), are:

$$R_{E1} = \exp\left[I_3 \left(\not{S}_1 + \not{S}_2 \right) / 2 \right] \;=\; \exp\left[I_3 \not{S} / 2 \right] \tag{36}$$

and:

$$R_{E2} = \exp\left[\left(\not{S}_1 + \not{S}_2 \right) I_3 / 2 \right] \;=\; \exp\left[\not{S} I_3 / 2 \right] \tag{37}$$

The vector action is the integral of the sum of the vector momentum integrated over the wavelength, thus:

$$\not{S}_1 + \not{S}_2 \;=\; \not{S} = \oint_{\lambda} \not{P} dt = \frac{mc}{\hbar} ct \rightarrow 1 \tag{38}$$

The product of these two rotors Eq.(36), and Eq.(37), is not a null and thus the exponent is not a Lorentz null without an initial phase adjustment between the photons.

The function is:

$$\bar{\Theta}_E = \rho R_{E1} R_{E2} \tag{39}$$

R_{E1} and R_{E2}, are time reversed rotors, and the product has only the three space vector momentum, not the time energy terms.

Adding an initial phase shift vector to one of the photons as noted earlier by way of $I\sigma^3\pi/2$ in the exponent. This gives an initial vector difference between the vector actions.

$$\bar{\Theta}_E = \left(\rho^{1/2}R_{E1}\right)I_3\sigma^3\left(\rho^{1/2}R_{E2}\right) \quad = \quad \left(\rho^{1/2}R_{E1}\right)\left(\rho^{1/2}\tilde{R}_{E2}\right)I_3\sigma^3 \qquad (40)$$

This phase shift of the photons in the particle makes the Systemfunction a Lorentz invariant and becomes very important to the proposed model shown below.

Proposed Physical Electron Model

It is the interaction of the electric vectors between the photons that produces a binding force and reduces the spin one to a half spin particle.

The concept of a physical model is somewhat alien to QM, but the Bohr atom has been useful in understanding the atomic processes, and the math constructs of GA makes The Bohr Model as well as photon model more understandable. As in the Bohr model where classical paths of the electrons defined the classical model, it is presumed that classical paths for bound photons can define the constituents of the Electron. In this case it is the photon structure and vectors that are defined by GA.

It is proposed that a pair of photons moving in opposite directions $\not{S}_1$ and $\not{S}_2$, as discussed above move in an orbit around their common center of mass Fig.1. Their oppositely aligned rotating electric vectors providing a binding force between the photons Fig 2.

Figure 1

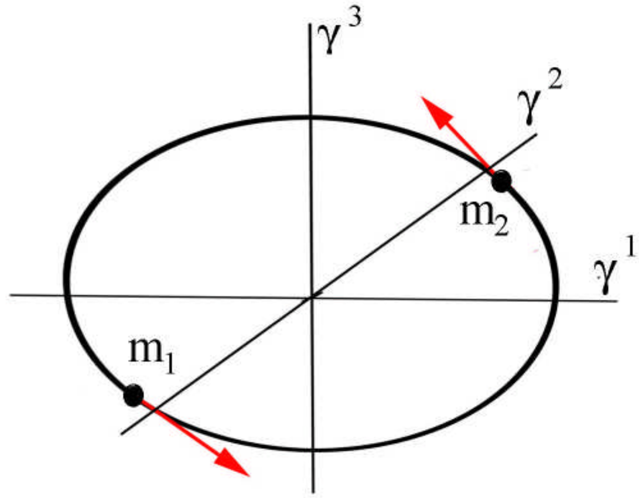


Figure 1, illustrates the concept for Photons 1 and 2 in an orbit in the $\gamma^1\gamma^2$ plane, with the mechanical angular momentum of their momentum around the γ^3 axis and the internal angular momentum of the electromagnetic vectors also along the γ^3 axis

Figure 2.

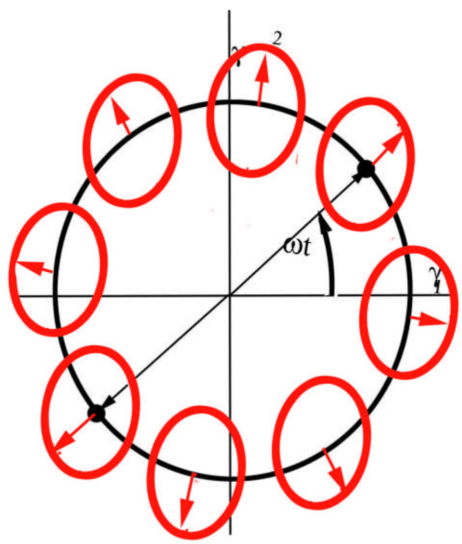


Figure 2, illustrates the concept of the rotating photon vectors of photon 1 and 2 in an orbit in the $\gamma^1\gamma^2$ plane, with the mechanical angular momentum of their mass also along the γ^3 axis

Electric Charge

From [3] the vector action defined in the Eq., are clearly definitions of the electric vector and that will be the presumption here. The magnetic vectors are easily included, but have been left out of this presentation for simplicity.

The model proposed asserts a pair of photons revolving around a common center. The action in addition to the internal angular momentum includes the angular momentum of the mechanical orbit.

$$S \sim \not{S}_{F1} + \not{S}_{O1} + \not{S}_{F2} + \not{S}_{O2} \tag{41}$$

:

With $\not{S}_1$ and $\not{S}_2$ being the sum of the photon internal action and the mechanical momentum of the orbit

$$R_{E1} = e^{-\left[I_3\left(\not{S}_1 + \not{S}_2\right)/2\right]} \rightarrow e^{-\left[I_3\left(\not{S}_{F1} + \not{S}_{O1} + \not{S}_{F2} + \not{S}_{O2}\right)/2\right]}, \tag{42}$$

With the photons orbiting a common center and the internal angular momentum aligned with the mechanical angular momentum, it is proposed that the mechanical phase of the orbit be in phase with the electric vector. Since the particle velocity is c this can only take place if the radius of the orbit is $\lambda\!\!\!^{-} = \lambda / 2\pi$, . The wavelength of the photon is: λ

Writing the actions in terms of the angle reference with the particle center of mass including the phase shift included making the Electron Systemfunction Lorentz invariant gives:

$$S \sim \left(\omega_{F1}t + \omega_{O2}t + \pi + \omega_{O1}t + \omega_{F2}t\right) \tag{43}$$

As noted the π phase shift anti-aligns the radial vectors such that:

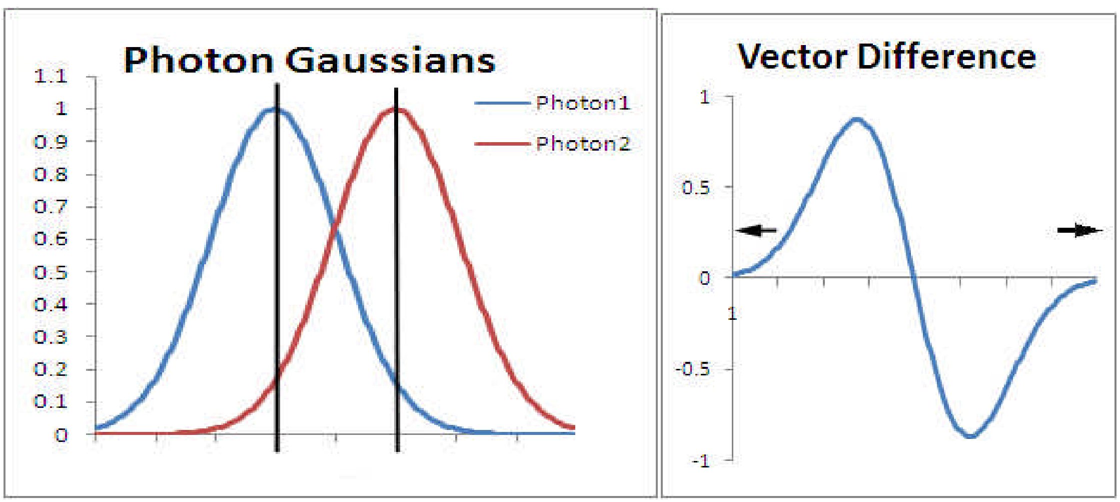
$$S \sim \left(\omega_{F1}t + \omega_{O2}t\right) - \left(\omega_{O1}t + \omega_{F2}t\right) \tag{44}$$

With all the vectors anti-aligned the sum of the vector action would be zero except for the fact that the photons are not at the same position, and their envelope, defined by a short range Gaussian Eq.(19), has a magnitude outside the confines of the particle.

From the model, (Fig 1), the two photons are revolving about their center of mass with anti-aligned electric vectors in phase with the rotation. As the particles rotate around the center of mass the electric vectors are always in a positive radial direction i.e.:

A Gaussian spatial relation associated with each photon defined by Eq.(19), assures the vectors are only null at the center of mass.

Figure 3 &4.

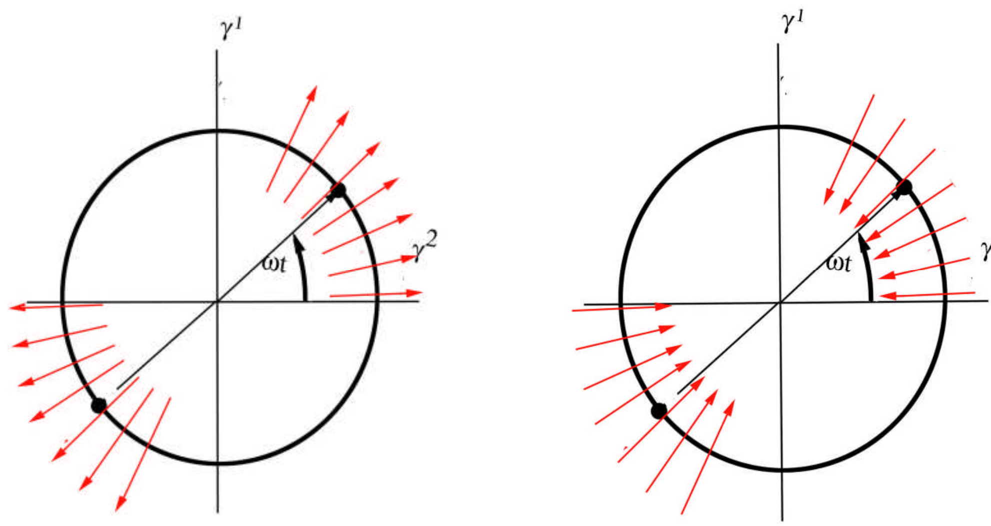


The electric vectors at one of the revolving photon are opposite to the electric vector of the other, and thus provide the binding forces between the photons. The value of the electric field is zero at the center of mass in contrast to the classical electron have an infinite field at the center.

The short range of electric vector inside Gaussian kernel is obviously not the source of the electric vector outside the photons orbit, but the Path Integral formulation of QFT, asserts the presence of the photon over all possible paths can provide the $1/r^2$ coulomb forces to external charges.

Charge Sign

Two photons orbiting in phase with the rotation can be either in phase or out with a phase difference of π in the rotational angle.



This is the mechanism defining the positive and negative charged particles.

Magnitude of Binding Force

As the photons rotate the electric vectors are opposite creating an attractive binding force between the opposite photon. The force is equivalent to the centrifugal force and the magnitude of the force can be evaluated

$$F_c = \frac{m_1 v^2}{r} = \frac{m_1 c^2}{\lambdabar_1} = \frac{m_1 c}{\lambdabar_1 \hbar} c\hbar = \frac{c\hbar}{\lambdabar_1{}^2} \sim \frac{1}{\alpha}\frac{Q^2}{r^2} \qquad (45)$$

This is equivalent to the force 137 times greater than a charge-charge binding force at the orbital diameter $2\lambda_1$

From a QFT standpoint these photon paths are the sum of all action paths and the two photons are being held together by the exchange of photons. It is thus asserted that this is a classical model of the entangled paths of two spin one vector bosons.

Solution to the Dirac & KG Solutions

At the center of mass, the square of the action is zero or at least constant and thus ρ can be considered constant, thus the vector function is:

$$\bar{\Theta}_E = e^{-\left(\not{S}^2 + 1 + I_3 \not{S} + I_3 \not{S}\right)/2} \tag{46}$$

The instantaneous four-derivative of the action is:

$$\not{\partial}\not{S} = \not{P} \tag{47}$$

From Eq.(11):

$$\left|\not{P}\right| = m_0 \tag{48}$$

From the same arguments made for Eq.(23):

$$\not{\partial}\,\not{S}^2 = 0 \tag{49}$$

With those conditions the electron Systemfunction is easily shown to be a solution to the Geometric equivalent K-G and Dirac expressions.

$$\bar{\Theta}_E = e^{-\left(\not{S}^2 + 1 + I_3 \not{S} + I_3 \not{S}\right)/2} \tag{50}$$

The first four derivative of the function of is the vector:

$$\not{\partial}\;\bar{\Theta} = I_3 \not{P}\,\bar{\Theta}, \tag{51}$$

which has a magnitude of:

$$\left| I_3 \not{P} \right| = m_0 \tag{52}$$

.

Taking the second derivative with the assumption of Eq.(49), gives:

$$\not{\partial} \not{\partial} \bar{\Theta} = \not{P}^2 \bar{\Theta} = m_0^2 \bar{\Theta} \tag{53}$$

Thus the electron and positron Systemfunctions are solutions to the vector equivalent Dirac and Klein Gordon expression.

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Spin

The spin of the particle is the time derivative of the rotor Eq.(36), thus:

$$\frac{d}{dt} R_{E1} = \frac{d}{dt} \exp\left[I_3 \not{S} / 2 \right] = \left(\frac{1}{2} \hbar \right) R_{E1} \sigma^3 , \tag{54}$$

This is identifiable as the spin for the electron moving along the γ^3 axis. It is not an expectation value, as would be the case in QM, but a property of the electron Systemfunction.

Proposed Neutrino Model (*speculation)*

There is little difference between the Systemfunction for an electron and a possible particle having the attributes of a neutrino. Starting from the function for an electron Eq.(27):

$$\bar{\Theta}_N = Ae^{-\left[\left(\not{S}_1 + \not{S}_2\right) + I_3\right]^2 / 2}, \tag{55}$$

Instead of two photons moving in opposite directions is proposed that the two photons are going along the same trajectory. The commutator of the two null vectors $\not{S}_1$, and $\not{S}_2$, i.e. the dot product, which constitute the Lorentz invariant rest mass for the Electron is zero.

$$\not{S}_1 \bullet \not{S}_2 = 0 \tag{56}$$

This eliminates the Lorentz invariant rest mass of the particle and gives the expanded Systemfunction for the Neutrino to be.

$$\bar{\Theta}_N = (\exp)\,\frac{-\left[\left(\not{S}_1^{\,2} + \not{S}_2^{\,2}\right) + 1 + I_3\not{S} + \not{S}I_3\right]}{2} \tag{57}$$

From the previous development of the electron this is seen as two photons coupled together having ½ spin a mass component associated with I_3^2 of. There is no rest mass per se but there is a fixed invariant mass associated with the ½ spin.

Selecting the event $\not{S}_1$, at which t and x_1 are zero the Gaussian exponent becomes

$$\frac{-\left(0 + \not{S}_2^{\,2}\right) - 1}{2} = 0 \tag{58}$$

The Gaussian has a maximum at $\not{S}_1$ if the second particle is at a specific relative interval as determined by:

$$-\left(\frac{x_2}{\lambda_2}\right)^2 + 1 = 0 \tag{59}$$

That is the position of $\not{S}_2$ is ahead of $\not{S}_1$ by one wavelength the same separation as in the electron, accept that it is along a linear trajectory rather than around an orbit.

The photons are thus moving along with locked positions one wavelengths apart with a combine spin of ½ and an invariant mass that is not a rest mass nor connected to the energy of the photons..

This model of a neutrino looks plausible, but should be taken as a bit speculative.

Conclusion

This paper presents a plausible complete GA model of photons, electrons, positrons, and the electron-positron pair annihilation process, and provides a platform for as well as a structure for other particles. The purpose is to provide to more mechanical view of QM, which could lead to a QFT formulation of the electron without infinities associated with a point particle..

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