Scalar Gravitational Theory with Variable Rest Mass

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ABSTRACT

In this paper we will present the mechanical dynamics of a gravitational system resulting from a specific, rest mass, scalar potential relation, that is equivalent in predicting orbital and photon motion to that of General Relativity in the weak field solutions. Particle motion and mass are defined by the summation of the effect, and motion, of the entirety of the other particles in the system, rather than a Riemannian spacetime structure. The value of the mass is not significantly different from that of a particle defined in a stationary asymptotically flat GR spacetime, when the dealing with point particles (Komar mass), and this is probably the reason the orbital equations from this theory are similar to the orbital equations in GR. The significant difference, however is that there is no stress energy tensor no energy assigned to a static field, and no black holes.

INTRODUCTION

Observations made in the weak solutions of General Relativity do not appear to be contradicted, and in this range the physical difference may not be measurable. The most significant difference between this development and GR is that in this case, there is conservation of energy of a photon moving in a gravitational potential. After taking account of the rest mass change at different elevations, the results of the Pound-Rebka-Snider experiment would indicate that the energy of the photon is conserved, thus photons can always escape a gravitational potential. In the interior of a neutron star with levels of the gravitational potential approaching one, the rest mass approaches zero, the velocity of light approaches zero, and there is total conversion of mass to energy.

The consequences result, for large masses, not in the prediction of black holes, but rather mass to Gamma ray converters. In a massive neutron star total conversion would create an energy engine suitable for driving unstable gamma ray bursters. The theory could suggest that the anomalous defined
gamma ray sources emissions of the galactic center, imaged by the ESA/INTEGRAL spacecraft could be from bodies close to the maximum mass [3]. The determination of the value of the maximum stellar mass will be important in determining the validity of this theory.

It should be clarified that Scalar Theory in this case refers to the fact that the instantaneous gravitational potential is a linear sum, defined as:

$$\phi = \sum_{n}^{N} \frac{2\mu_{n}}{r_{n}},$$  \hspace{1cm} (1)

and not a reference to the potential functions $\Box^{2}\phi = 4\pi G\varepsilon_{\rho}$, and as such the scope of this paper does not include wave motion, or gravitational radiation.

In an earlier paper we presented the relation between the observable mass and what we have designated as the gravitationally field free mass [1]:

$$M_{f} = M / \alpha$$ \hspace{1cm} (2)

Where $\alpha$ is a scalar variable, and related to the gravitational interaction by:

$$\alpha = \left( 1 - \sum_{n}^{N} \frac{\alpha_{n}\mu_{n}}{r_{n}} \right).$$ \hspace{1cm} (3)

We will take Eq.(2) to be the defining relation for the motion of the particles in the system, and presume that the total energy of a particle in a conservative system to be constant.

**GENERAL DEVELOPMENT**

First noted is that the sum in Eq. (3) is a function defined at all points in the system, and defines the value of the mass for the particles in empty space. Although points may have the same value of alpha, they not necessarily have the same configuration. Expression (2) for the rest mass of a particle is:

$$M_{r}^{2} = \frac{M_{f}^{2}}{\alpha_{f}} = \frac{M_{20}^{2}}{\alpha_{2}}$$ \hspace{1cm} (4)
where the 1 and 2 subscripts represent the same particle in two positions. And \( M_F \) is the mass independent of the effect of any other particles. The zero in the subscript implies the rest momentum.

If Eq. (4) is true then, since there can be internal motion in a mass of particles, which would not be distinguishable from the rest mass by an external observer, the relativistic masses must have the same proportion. i.e.:

\[
\frac{M_{10}^2}{\alpha_1^2} = \frac{M_{20}^2}{\alpha_2^2} \quad \rightarrow \quad \frac{M_1^2}{\alpha_1^2} = \frac{M_2^2}{\alpha_2^2}
\]  

(5)

Since the rest, and relativistic masses are related by:

\[
\frac{M_{10}^2}{\alpha_1^2} = \frac{M_1^2}{\alpha_1^2} \left[ 1 - \frac{v_1^2}{c_1^2} \right] \quad \frac{M_{20}^2}{\alpha_2^2} = \frac{M_2^2}{\alpha_2^2} \left[ 1 - \frac{v_2^2}{c_2^2} \right]
\]  

(6)

then \( c \) and \( v \) have to be proportional, or:

\[
\left( \frac{v}{c} \right)_1 = \left( \frac{v}{c} \right)_2
\]  

(7)

From Eq. (3), and Eq. (7), the ratio for two configurations for a single rest particle, one in free space (1), and one in propinquity to a local gravitating mass (2), is:

\[
\frac{\alpha_1^2}{\alpha_2^2} = \frac{M_{10}^2}{M_{20}^2} = \frac{\left( \frac{1}{1 - \sum \alpha_m \frac{\mu_m}{r_m}} \right)^2}{\left( \frac{1}{1 - \sum \alpha_m \frac{\mu_m}{r_m} - \alpha_i \frac{\mu_{\text{loc}}}{r_{\text{loc}}}} \right)^2} = \frac{1}{\left( 1 - \frac{\mu_{\text{loc}}}{r_{\text{loc}}} \right)^2},
\]  

(8)

where (loc) designates variables of the local gravitating mass. This presents the relationship between the rest mass as a function of the gravitational potential.

Since a moving particle experiences the “relativistic mass” of the gravitating mass \( \mu_{\text{loc}} \), in the event the particle, (2) in Eq. (8) is moving, the mass must be:
\[ M_{\text{loc}} = M_{0\text{loc}} / \sqrt{1 - v_{2}^2 / c^2} \]  \hspace{1cm} (9)

Then restating Eq.(8), we have:

\[ \frac{M_{l0}^2}{M_{2}^2 - M_{2}^2 (v/c)^2} = \left(1 - \frac{\mu_{\text{0loc}}}{r_{\text{loc}} \sqrt{1 - v_{2}^2 / c^2}}\right)^{-2} \]  \hspace{1cm} (10)

or:

\[ M_{l0}^2 c^2 \left(1 - \frac{\mu_{\text{0loc}}}{r_{\text{loc}} \sqrt{1 - v_{2}^2 / c^2}}\right)^2 = M_{2}^2 c^2 - M_{2}^2 v^2 \]  \hspace{1cm} (11)

And since we have:

\[ M_{2}^2 = M_{l1}^2 \frac{\alpha_{l1}^2}{\alpha_{2}^2} \]  \hspace{1cm} (12)

we can write Eq (11) as:

\[ M_{l0}^2 c^2 \left[1 - \frac{\mu_{\text{0loc}}}{r_{\text{loc}}} \left(1 + \frac{1}{2} v^2 \right)\right]^2 = M_{2}^2 c^2 - M_{2}^2 v^2 \left(1 - \frac{\mu_{\text{0loc}}}{r_{\text{loc}}}\right)^2 \]  \hspace{1cm} (13)

**Orbital Mechanics**

We now have a differential expression relating, the initial mass, the total mass, the velocity, and the distance to the local gravitating mass. We should thus be able to solve for the orbital motion, without need to make assumptions about the force mass relation, or the influence of a vector potential on the test particle. We have only assumed the rest mass relation between particles, and the relativistic mass velocity relation.

In the following it will be shown that the equations of motion defined by Eq. (13) produces orbital relations, equivalent to the weak field GR relations, with the same perihelion advance:

Rearranging we have:
\[
\frac{\left(\text{Mc}\right)_{10}^2 - \left(\text{Mc}\right)_{2}^2}{\left(\text{Mc}\right)_{10}^2} c^2 - 2 \left(1 + \frac{1}{2} \frac{v^2}{c^2}\right) \frac{\mu_{\text{loc}} c^2}{r_{\text{loc}}} + \frac{M^2_1}{M^2_{10}} v^2 \left(1 - 2 \frac{\mu_{\text{loc}}}{r_{\text{loc}}} \right) = 0 \tag{14}
\]

Noting in a conservative system the first term in Eq. (14) is constant, that is the total mass, and the initial rest mass are constant, we can rearrange, and write this in the usual form:

\[
2 \epsilon = -2 e^2 \frac{\mu_{\text{loc}}}{r_{\text{loc}}} + v^2 \left(1 - 3 \frac{\mu_{\text{loc}}}{r_{\text{loc}}} \right) = 0 \tag{15}
\]

Replacing \(r\) with \(u = 1/r\), and dropping the subscripts we have:

\[
2 \epsilon = -2 \mu u c^2 + v^2 (1-3 \mu u) = 0 \tag{16}
\]

or:

\[
2 \epsilon = -c^2 2 \mu u + \left(\frac{dr}{dt}\right)^2 (1-3 \mu u) + u^2 \left(r^2 \theta^2\right) (1-3 \mu u) = 0 \tag{17}
\]

using the relations:

\[
h = \left(r^2 \theta^2\right) \quad \left(\frac{dr}{dt}\right)^2 = h^2 \left(\frac{du}{d\theta}\right)^2 \tag{18}
\]

\[
\frac{d}{d\theta} \left(u^2 h^2 - 3 \mu u^2 h^2\right) = 2 u h^2 \frac{du}{d\theta} - 9 \mu h^2 u^2 \frac{du}{d\theta}
\]

and noting that:

\[
\frac{d}{d\theta} \left[ \left(\frac{dr}{dt}\right)^2 (1-3 \mu u) \right] = 2 h^2 \left(\frac{du}{d\theta}\right) \left(\frac{d^2 u}{d\theta^2}\right) (1-3 \mu u) - 3 \mu \left(\frac{dr}{dt}\right)^2 \left(\frac{du}{d\theta}\right) \tag{19}
\]

The differential with respect to \(\theta\), is:

\[
\begin{bmatrix}
-2 \mu e^2 \frac{du}{d\theta} + 2 h^2 \frac{du}{d\theta} \frac{d^2 u}{d\theta^2} (1-3 \mu u) \\
-3 \mu h^2 \left(\frac{du}{d\theta}\right) \left(\frac{du}{d\theta}\right)^2 + 2 h^2 u \frac{du}{d\theta} - 9 \mu h^2 u^2 \frac{du}{d\theta}
\end{bmatrix} = 0 \tag{20}
\]
Factoring: \( 2h^2 \frac{du}{d\theta} \) we have:

\[-\frac{c^2 \mu}{h^2} + \frac{d^2u}{d\theta^2}(1 - 3\mu u) - \frac{9}{2} \mu u^2 + u - \frac{3}{2} \left( \frac{du}{d\theta} \right)^2 \mu = 0 \]

or:

\[\frac{d^2u}{d\theta^2} + \frac{u}{(1 - 3\mu u)} - \frac{u_0}{(1 - 3\mu u)(1 - e^2)} - \frac{3}{2} \mu u^2 + \frac{9}{2} \mu u^2 \mu \mu u_0 - \frac{3}{2} \mu \left( \frac{du}{d\theta} \right)^2 \frac{\mu}{(1 - 3\mu u)} = 0 \]

where:

\[\frac{u_0}{1 - e^2} = \frac{c^2 \mu}{h^2} \]

and \( e \) is the eccentricity.

Taking out terms in second order of \( \mu \), then this is:

\[\frac{d^2u}{d\theta^2} + \frac{u}{(1 - e^2)} - \frac{u_0}{(1 - e^2)} - \frac{3}{2} \mu u u_0 + 3 \mu u^2 - \frac{9}{2} \mu u^2 - \frac{3}{2} \mu \left( \frac{du}{d\theta} \right)^2 = 0 \]

The orbital equation is then:

\[\frac{d^2u}{d\theta^2} + \frac{1}{(1 - e^2)} \left[ \frac{3}{2} \mu u^2 + \frac{3}{2} \mu u_0 \left( \frac{2u}{(1 - e^2)} + e^2 \frac{u_0}{(1 - e^2)} \sin^2 \theta \right) \right] = 0 \]

which is not exactly the Einstein orbital equation, but does have the same perihelion advance.

The perihelion advance factor, by the procedure of Robertson Noonan [4] is:

\[\sigma = \frac{1}{2} \frac{d}{du} \left[ \frac{3}{2} \mu u^2 + \frac{3}{2} \mu u_0 \left( \frac{2u}{(1 - e^2)} + e^2 \frac{u_0}{(1 - e^2)} \sin^2 \theta \right) \right] \bigg|_{u = u(0)} \]

or
\[ \sigma = \left[ \frac{3}{2} \mu u + \frac{3}{2} \frac{\mu}{u_0} \left( \frac{1}{1-e^2} + e^2 \frac{u_0}{1-e^2} \cos \theta \sin \theta \frac{d\theta}{du} \right) \right]_{u=u(0)} \], (27)

or

\[ \sigma = \left[ \frac{3}{2} \mu u + \frac{3}{2} \frac{\mu}{u_0} (1 + e \cos \theta) \right]_{u=u(0)} = \frac{3\mu}{p} \], (28)

which compares within experimental error with a more exact value of the Einstein perihelion advance by Powell [5]:

\[ \frac{3\mu}{p} \left[ 1 + \frac{3\mu}{p} \frac{(1+e)^2}{2e} \right] \], (29)

**Orbital Deflection**

For the deflection of a particle traveling near light speed can evaluate the orbital Eq, (24) by setting:

\[ u_n = \frac{\mu}{R^2}, \quad u = \frac{\cos \theta}{R}, \] (30)

for small \( \mu \) terms: This yields a deflection of:

\[ \frac{5}{2} \mu \] (31)

or a total angular deflection of:

\[ \frac{5\mu}{R} \] (32)

Which is greater by 25% that the known angular light deflection of:

\[ \frac{4\mu}{R} \] (33)

The high speed particle deflection and the light deflection developed below, unlike in the prediction of GR are not the same. This is not presumed to be a fatal flaw since, the rules of motion for a particle and a photon in the scalar theory are not the same. As a mass particle enters a gravitational potential from an external observer, its velocity accelerates, whereas the velocity for the photon decelerates, (Shapiro effect). At this point there has not been a
measurement of high-speed particle deflection, and thus no available comparison. Unless there is an undiscovered component, or calculation error, the difference in the mass particle deflection would provide a testable distinction with GR.

**Photon Energy**

From the defining relation of this theory Eq. (2), the view of the Pound-Rebka-Snider, Mossbauer effect experiment (1960–1965)[6] changes. Instead of the photon losing energy as the photon rises in the tower, the emission of the photon at the bottom of the tower, is from a less massive generator, and at a lower frequency. The generated frequency plus the added Doppler frequency provided by the velocity of the source in the experiment equals the frequency at the top, thus the photon loses no energy in the flight up the tower.

\[
v_{D}(1 - \frac{\mu}{r}) + v_D - v_T.
\]

This is a major departure from General Relativity, but does not have a locally measurable difference.

**Proper Deflection and Velocity of Light**

In order to accommodate the change in the rest mass associated with this theory, (Eq. (8)) there are other consequences, that can be deduced, that happen as the result, the most notable is the change in the velocity of light. The purpose of the following exercise is to determine the change in value of the relative velocity of light inside a gravitational field, as observed by an external observer.

To that end we will make a few assumptions that are consistent the theory.

**Assumptions**

1) The frequency of an atom, and thus extended to an atomic clock, is proportional to the rest mass. This assumption is equivalent to the potential time dilation of General Relativity, however in this case, the change is the result, not of a change in the time scale, but a change in the rest mass.
2) The physical dimensions of a material object at rest are invariant in a gravitational potential.

In order to determine the speed of light shift in a gravitational potential we will use a thought experiment based on the above assumptions. One can take the results either way. The results lead to the assumptions, or the assumptions lead to the results.

1) An laser interferometer is set up in an elevator on the top floor of a building, with a standing wave, having an integral number of wavelength across a resonating cavity.
2) The apparatus is lowered to the bottom floor.
3) We will make the assumption that there is no observable difference in the number of standing waves in the resonating cavity. This is also required by the equivalence principle

Using our assumptions, and Eq. (8) the frequency has decreased as a result of the decreased rest mass of the system at the lower position, and is:

$$v = v_0 \left(1 - \frac{\mu_{loc}}{r_{loc}}\right) \quad (35)$$

If the frequency has declined by the potential factor then the wavelength would extend beyond the interferometer space, if there were not an equivalent reduction of the wavelength by the same factor.

$$\lambda = \lambda_0 \left(1 - \frac{\mu_{loc}}{r_{loc}}\right) \quad (36)$$

Since the product of $v\lambda$ is the velocity of light, we have for a change in the velocity:

$$c = \lambda_0 v_0 \left(1 - \frac{\mu_{loc}}{r_{loc}}\right) \left(1 - \frac{\mu_{loc}}{r_{loc}}\right) = c_0 \left(1 - \frac{\mu_{loc}}{r_{loc}}\right)^2 \quad (37)$$

We know this is the proper value or relative velocity by way of the measured Shapiro effect. This relative velocity also leads to the proper, deflection of light, in agreement with GR by way of Fermat’s principle, as shown by Blandford et al [7].
Momentum Considerations

Although the energy of a photon rising or falling in a gravitational potential is unaffected the same cannot be true of the momentum. With momentum given by:

\[ p = \frac{E}{c} = \frac{h}{\lambda} = \frac{h}{\lambda_0 \left( 1 - \frac{\mu_{\text{loc}}}{r_{\text{loc}}} \right)^{-1}}. \]  \( (38) \)

Thus from the perspective of this theory, for both the photon, and a massive particle, on entering a gravitational potential, the total energy remains constant, but the momentum increases.

CONCLUSION

With simple assumptions regarding the relation between rest mass and distance, proper gravitational dynamics and phenomena can predicted. The proposed theory yields the proper orbital equations, with the proper perihelion advance, deflection of light, and gravitational red shift. The gravitational potential exchanges no energy with photons, thus photons cannot be bound in a gravitational field, and the existence of a black hole as defined in GR would not be possible.

1 DT Froedge, The Origin of the Klein-Gordon-Dirac expression, and its implication in QM and Particle Mass Ratios: http://www.arxdtf.org/


