The Concept of Mass as Interfering Photons
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Foreword
The more descriptive, but simplistic presentations of concepts contained in this paper are extracted from the more detailed aspects of two other papers, [1], and [2].

Abstract
For most purposes in physics the concept of mass particles and photons are treated as though they are completely separate and distinct entities having little connection accept through collision interactions. This paper explores the concept that a mass particle can be viewed as a pair of trapped photons in a mass-less box, for both dynamic, gravitational, and annihilation considerations.

Introduction
The special theory of relative, through the Lorentz transformations, yields the energy velocity relation for both photons and particles, one through a shift in frequency, the other through a shift in mass. Considering these particles as different forms of energy, however, bestows a distinction between the forms of energy that is possibly unwarranted. The Lorentz transforms applied to a pair of localized photons can be shown to yield the same results as the transforms applied to a mass particle. If particles are considered to be three-dimensional interference patterns of two photons that are somehow spatially constrained to their center of mass, the properties match very well the dynamics of real mass particles. One may not subscribe to the substance of this, but it does give a useful perspective regarding mass, rest mass, and energy, and offer an intuitive understand of why particles cannot exceed the speed of light.

1 Momentum
Consider a thought experiment, in which two photons are placed in a perfectly reflecting massless container. Presuming that if the two photons are not aligned in the given frame, there has to be some sub-light speed frame of reference, in which the photons are aligned, and in opposite directions, as well as having equal energy and frequency. This frame is thus the rest frame for the center of mass for the two photons

Using the momentum for the photons to be:
\[ \vec{p} = M\vec{c} = \left[ \frac{\hbar \nu}{c} \right] \vec{c}, \quad (1) \]

where we can designate a “mass” for the photon to be \( M = \hbar \nu / c^2 \). The momentum of the container with respect to a moving frame of reference with velocity \( v \) is then:
\[ \vec{P} = (M_1 + M_2)\vec{v}. \quad (2) \]

From the perspective of the individual opposite-going photons the momentum is:
\[ P = P_1 + P_2 = \frac{\hbar \nu_1 - \hbar \nu_2}{c} = \frac{\hbar \Delta \nu}{c} = \frac{\hbar}{\lambda_B}. \quad (3) \]

The wavelength of the difference in the frequency here, or the “beat” frequency, is just the simple deBroglie wavelength.

The total energy, which is the sum of the energy of the photons, and thus sum of the frequencies, yields the simple Compton wavelength:
\[ \frac{E_1 + E_2}{c} = \frac{\hbar \nu_1 + \hbar \nu_2}{c} = \frac{\hbar}{\lambda_C}. \quad (4) \]

Using the above noted designation for “mass” we can write for the total “mass”:
\[ M_r = (\hbar \nu_1 + \hbar \nu_2) / c^2, \quad (5) \]

and
\[ P = M_r v = (M_1 - M_2) c. \quad (6) \]

Solving for velocity:
\[ \frac{v}{c} = \frac{(M_1 - M_2)}{(M_1 + M_2)}. \quad (7) \]

For this to conform to a real relativistic particle the form has to be:
\[ M_0^2 = M^2 \left[ 1 - \left( \frac{v}{c} \right)^2 \right]. \quad (8) \]

Putting in \( M_r \), and \( v/c \) and solving gives:
\[ M_0^2 = (M_1 + M_2)^2 - (M_1 - M_2)^2 = 4M_1M_2. \] (9)

So the square of the rest mass of the particle, is four times the product of the “mass” of the individual photons.

\section*{II Doppler}

Now for the moment, we can shift gears, and look at this same picture from the standpoint of the Doppler shift, on transformation of velocity coordinates for the two photons.

The relativistic Doppler shift of the photons from one velocity frame to another is:

\[ \begin{align*}
\nu_1' &= \nu_1 \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \\
\nu_2' &= \nu_2 \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}
\end{align*} \] (10)

or using the above noted conventions for mass:

\[ \begin{align*}
M_1' &= M_1 \left[ 1 - \frac{v}{c} \right] \\
M_2' &= M_2 \left[ 1 + \frac{v}{c} \right]
\end{align*} \] (11)

Multiplying the two relations gives:

\[ M_1'M_2' = M_1M_2 = \text{constant} \] (12)

and simple math gets:

\[ M_1M_2 = \frac{(M_1 + M_2)^2 - (M_1 - M_2)^2}{4}, \] (13)

and:

\[ 1 - \left( \frac{v}{c} \right)^2 = \frac{4M_1M_2}{(M_1 + M_2)^2} = \frac{M_0^2}{M^2}, \] (14)

which is the same as the above relation, found for conformance to relativistic kinematics, the model thus transforms properly.
Scalar Gravitational Considerations

The interfering photon concept can be extended to gravitational effects by way of including considerations from the paper on Scalar Gravitation [1], which gives the relation between the velocity of light, the rest mass, and the gravitational potential. From [1], we have:

\[
\frac{M_{20}^2}{M_{10}^2} = \left(1 - \frac{\mu}{r}\right)^2 \quad \frac{c}{c_0} = \left(1 - \frac{\mu}{r}\right)^2
\]  

(15)

Where \( M_{10} \) is the free rest mass and \( M_{20} \) is the rest mass inside the potential. The gravitational effect influences the interfering particle by way of a slow down in the speed of light with respect to an external observer. This slow down in \( c \) produces a decrease in the rest mass, which in turn, conservation of momentum requires an increase in the velocity of the particle, observed as gravitational attraction.

This development will be in reverse order for the purpose of connecting to the previous material. That is, we will assume the form of the modification of the photons due to the gravitational field, and show that it leads to the relations, Eqs. (15), rather than deduce the results from Eqs. (15). There is a small second order approximation, Eqs. (28), making the relation only approximate, but the difference is not measurable.

We can represent the four-momentum of the opposite going photons previously defined for the particle in the Clifford-Dirac matrix form as:

\[
\vec{P}_1 = \gamma_1 M_1 c_x + \gamma_2 M_1 c_y + \gamma_4 (M_1 c_z) \\
\vec{P}_2 = -\gamma_1 M_2 c_x - \gamma_2 M_2 c_y - \gamma_4 M_2 c_z + \gamma_4 (M_2 c)
\]  

(16) \hspace{1cm} (17)

The magnitude of each of these four-momentum is zero for covariance, and the sum of two such moments must be constant.

For conformance to the Scalar Theory gravitational consideration given in [1], we will postulate that the modification of the momentum must be:

\[
\vec{P}_i = [\gamma_1 M_i c_x + \gamma_2 M_i c_y + \gamma_4 M_i c_z] \left(1 + \frac{2\mu^2}{r^2} \right) + \gamma_4 (M_i c)
\]  

(18)

and
\[
\vec{P}_s = \left[ -\gamma_1 M_2 c_x - \gamma_2 M_1 c_x - \gamma_4 M_3 c_z \right] \left( 1 - \frac{2\mu}{r} - \frac{\mu^2}{r^2} \right) + \gamma_4 (M_3 c) .
\] (19)

Which is equivalent to the addition of the space-like momentum to the two opposite going photons, one increasing and one decreasing their respective photons. \( P_1 \) is the infalling photon, and is being increased. The momentum modifications are then:

\[
P_{G1} = \left[ +\gamma_1 M_1 c_x + \gamma_2 M_1 c_x + \gamma_4 M_1 c_z \right] \left( \frac{2\mu}{r} - \frac{\mu^2}{r^2} \right) \]
(20)

\[
P_{G2} = \left[ +\gamma_1 M_2 c_x + \gamma_2 M_2 c_x + \gamma_4 M_2 c_z \right] \left( \frac{2\mu}{r} - \frac{\mu^2}{r^2} \right),
\] (21)

or for the square of the magnitude:

\[
\left| \vec{P}_1 + \vec{P}_{G1} + \vec{P}_2 + \vec{P}_{G2} \right|^2 = \text{constant}
\]

The magnitude of the sum of two four-moments can then be written:

\[
\left( \vec{P}_1 + \vec{P}_2 \right) = \left[ \gamma_1 (M_1 - M_2) c_x + \gamma_2 (M_1 - M_2) c_x + \gamma_3 (M_1 - M_2) c_z \right] + \gamma_4 (M_1 + M_2) c
\]

\[
+ \frac{c}{r} \left[ \gamma_1 (M_1 + M_2) r_x + \gamma_2 (M_1 + M_2) r_x + \gamma_3 (M_1 + M_2) r_z \right] \frac{2\mu}{r} - \frac{\mu^2}{r^2}
\] (22)

In effect the momentum of a pair of virtual photons has been added to the particle from the perspective of the external world. This is effectively because the velocity of light, which has been slowed in the potential, causes a reduction in the rest mass, which in turn reduces the momentum. For the four-momentum to remain constant, the additional photon momentum must be added. This photon momentum is necessary to keep the total energy and momentum constant with respect to the external world, and results in an acceleration of the particle.

We can manipulate this to become:

\[
\left( \vec{P}_1 + \vec{P}_2 \right) = (M_1 + M_2) c \left\{ \left[ \frac{(M_1 - M_2) c}{(M_1 + M_2) c} \right] + \frac{r}{r} \frac{2\mu}{r} - \frac{\mu^2}{r^2} + \gamma_4 \right\}
\] (23)

From the earlier relations Eq. (7)
\[ \frac{(M_1 - M_2)}{(M_1 + M_2)} = \frac{v}{c}, \quad (24) \]

Eq. (23) becomes:

\[ (\vec{p}_1 + \vec{p}_2) = -(M_1 + M_2)c \left\{ \frac{v}{c} + \frac{r}{r} \sqrt{2\mu \over r - \mu^2 \over r^2} + \gamma_4 \right\}, \quad (25) \]

or:

\[ |\vec{p}_1 + \vec{p}_2|^2 = (M_1 + M_2)^2 c^2 \left[ 1 - \left( \frac{v}{c} + \sqrt{2\mu \over r - \mu^2 \over r^2} \right)^2 \right] \quad (26) \]

or approximately:

\[ |\vec{p}_1 + \vec{p}_2|^2 \approx (M_{10} + M_{20})^2 c^2 \left[ 1 - \left( \frac{v}{c} \right)^2 \right] \left[ 1 - \left( \frac{\mu}{r} \right)^2 \right]. \quad (27) \]

Where we have used the approximation:

\[
\begin{align*}
\left[ \gamma_4 + \frac{r}{r} \left( \frac{v}{c} + \sqrt{2\mu \over r - \mu^2 \over r^2} \right) \right]^2 &= 1 - \left( \frac{v}{c} + \sqrt{2\mu \over r - \mu^2 \over r^2} \right)^2 \\
&= \left[ 1 - \left( \frac{v}{c} \right)^2 - 2\mu \over r + \mu^2 \over r^2 \right] - 2\frac{v}{c} \sqrt{2\mu \over r - \mu^2 \over r^2} \\
&= \left[ 1 - \left( \frac{v}{c} \right)^2 \right] \left[ 1 - \left( \frac{\mu}{r} \right)^2 \right] = \left[ 1 - \left( \frac{v}{c} \right)^2 \right] \left[ 1 - \sqrt{2\mu \over r - \mu^2 \over r^2} \right] \\
&= \left[ 1 - \left( \frac{v}{c} \right)^2 - 2\left( \frac{\mu}{r} \right) + \left( \frac{\mu}{r} \right)^2 \right] + 2\left( \frac{v}{c} \right)^2 \left[ 2\left( \frac{\mu}{r} \right) - \left( \frac{\mu}{r} \right)^2 \right] \\
&= \left[ 1 - \left( \frac{v}{c} \right)^2 - 2\left( \frac{\mu}{r} \right) + \left( \frac{\mu}{r} \right)^2 \right] + 2\left( \frac{v}{c} \right)^2 \left[ 2\left( \frac{\mu}{r} \right) - \left( \frac{\mu}{r} \right)^2 \right] \\
&= \left[ 1 - \left( \frac{v}{c} \right)^2 - 2\left( \frac{\mu}{r} \right) + \left( \frac{\mu}{r} \right)^2 \right] + 2\left( \frac{v}{c} \right)^2 \left[ 2\left( \frac{\mu}{r} \right) - \left( \frac{\mu}{r} \right)^2 \right] 
\end{align*}
\] \quad (28)

The error in the approximation being the difference of the last two terms.

Since the magnitude of the four-momentum is constant Eq. (27) becomes:

\[ |\vec{p}_1 + \vec{p}_2|^2 \approx (M_1 + M_3)^2 c^2 \left[ 1 - \left( \frac{v}{c} \right)^2 \right] \left[ 1 - \left( \frac{\mu}{r} \right)^2 \right] = M_0^2 c^2, \quad (29) \]
where the right side is the constant rest mass. From this we see that the, potential free, rest mass is:

\[
M_0 = \left(1 - \left[\frac{v}{c}\right]^2\right)M(r = \infty)
\]

\[
M_0 = \left[1 - \left(\frac{\mu}{r}\right)^2\right]M(v = 0)
\]

(30)

The presence or absence of the additional photon momentum, results in the same effect on the rest mass, as the presence or absence of the gravitational potential shown in the Scalar Theory. What is really being said, is that as the particle enters the gravitational potential with a decreased speed of light, the velocity has to speed up to keep the magnitude of the four moment constant.

One could also note that in Eq. (22) if \(M_1 = M_2\), when \(r \rightarrow \mu\), the momentum becomes just the momentum of a photon with energy equal to the sum of the energy of the two initial photons.

**Electron-Positron Annihilation**

Replacing \(\mu\) with the Compton radius for the electron:

\[
\frac{\alpha h}{M_c r} = \frac{\alpha_r}{r}
\]

(31)

in the above relations, give the proper Mechanical result for the electron-positron annihilation. That is, as two particles merge the velocity of each increases to \(c\) and become two opposite going photons. In a simplistic description as the pair merge, the left going photons in the two particles, and the right going photons in the two particles, constructively interfere giving two opposite going free photons. The process is a little more complicated for annihilation than gravitation, however since by QFT, the process is controlled incrementally by photon exchange, thus the angular momentum in Eq. (31), must be:

\[
M_c r = n h,
\]

(32)

Spin considerations for pair annihilation is treated in Appendix I.
Conclusion

The concurring points of similarity of the model and the particles are then:

1) The deBroglie wavelength.
2) The Compton wavelength
3) The zero velocity or rest mass
4) The total energy
5) Velocity transforms
6) Gravitational effects
7) Annihilation process

Using a reflecting container is somewhat artificial, but as in the case of the transformation of momentum between velocity frames, the gross mechanics do not depend on the internal structure. All of the real internal constraints such as spin, energy, etc, which may be important to the actual mechanics of holding a particle together are not necessary to the analogy.

The dynamics of the center of mass of the two photons is the same whether the photons are confined or not, accept in one case there is a mass particle. It is also easy to understand from this model why mass particles do not exceed the speed of light.


Appendix I

Spin Considerations of electron-positron annihilation

The set of electron coordinate matrix representation we normally use is:
\[
\gamma_{e1} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \gamma_{e2} = \begin{bmatrix} -i & i \\ i & -i \end{bmatrix}, \quad \gamma_{e3} = \begin{bmatrix} -1 & +1 \\ +1 & -1 \end{bmatrix}, \quad \gamma_4 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
\]

And the corresponding opposite moving positron is:

\[
\gamma_{p1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \gamma_{p2} = \begin{bmatrix} -i \\ i \\ i \end{bmatrix}, \quad \gamma_{p3} = \begin{bmatrix} -1 \\ +1 \\ +1 \end{bmatrix}, \quad \gamma_4 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}
\]

Multiplying this set by what we call the charge matrix \(-iS\): \(S = \gamma_1 \gamma_2 \gamma_3\)

\[
iS = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}
\]

We have the normal spin matrix for electrons and positrons:

\[
\sigma_{e1} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}, \quad \sigma_{e2} = \begin{bmatrix} -i & i \\ -i & i \end{bmatrix}, \quad \sigma_{e3} = \begin{bmatrix} 1 \\ +1 \end{bmatrix},
\]

and

\[
\sigma_{p1} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \sigma_{p2} = \begin{bmatrix} -i \\ i \\ -i \end{bmatrix}, \quad \sigma_{p3} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}
\]

Having \(\frac{1}{2}\) spin commutation relations:

\[
\sigma_x \sigma_y - \sigma_y \sigma_x = 2\sigma_z
\]

Readjusting we can write this as:

\[
\sigma_{e1} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}, \quad \sigma_{e2} = \begin{bmatrix} -i & i \\ -i & i \end{bmatrix}, \quad \sigma_{e3} = \begin{bmatrix} 1 \\ +1 \end{bmatrix},
\]

and

\[
\sigma_{p1} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \sigma_{p2} = \begin{bmatrix} -i \\ i \\ -i \end{bmatrix}, \quad \sigma_{p3} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}
\]
\[
\sigma_{p_1} = \begin{bmatrix}
1 & 1 & 1 \\
(1-1) & 1 & (1-1) \\
-1 & -1 & -1
\end{bmatrix}
\quad \sigma_{p_2} = \begin{bmatrix}
-i & (-i+i) \\
(i-i) & i & i
\end{bmatrix}
\quad \sigma_{p_3} = \begin{bmatrix}
-1 \\
-1 \\
-1
\end{bmatrix}
\]

The sum of these two sets are equal to a second pair which are:

\[
\sigma_1 = \begin{bmatrix}
-1 \\
-1 \\
-1
\end{bmatrix}
\quad \sigma_2 = \begin{bmatrix}
-i \\
i \\
-i
\end{bmatrix}
\quad \sigma_3 = \begin{bmatrix}
-1+i \\
-1+i \\
-1+i
\end{bmatrix}
\]

and

\[
\sigma_1 = \begin{bmatrix}
-1 \\
+1 \\
+1
\end{bmatrix}
\quad \sigma_2 = \begin{bmatrix}
-i \\
-i \\
i
\end{bmatrix}
\quad \sigma_3 = \begin{bmatrix}
-1-i \\
-1-i \\
-1-i
\end{bmatrix}
\]

Which are just the spins for opposite going photons having spin one and photon commutation relations:

\[
2\sigma_z = (1-i)(\sigma_x\sigma_y - \sigma_y\sigma_x)
\]

Thus we have shown that two coincident opposite going photons, have the equivalent spin as a coincident electron-positron pair.