# Particle Mass Ratios

D.T. Froedge

*Formerly Auburn University*
*Phys-dtfroedge@glasgow-ky.com*

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## Abstract

Based on the developments in a previous paper [1], this paper presents straightforward explanation of particle mass ratios, yielding specific values for some well studied particles. The additional nuclear modes postulated, are similar to the Schrödinger modes in the atom, and the mass ratios calculated for these particles have been calculated within the experimental values. The examined particles are the Proton, Neutron, Muon, Tauon, and Pion+/-/0**.**

## Introduction

When starting from an empirical approach to the mass ratios, the charge of numerology can be leveled, noting that relating mass ratios by arbitrary combinations of integers and functions of pi can generate lots of numbers. In addition accounting for small residuals differences with more parameters increases the validity of the charge. A careful examination of the presentation however, will show that there is a theoretical basis for the paper, and the probabilities of  arriving at the accuracies shown without a causal relationship is vanishingly small.

The function has 3 selectable numbers for the modes in the function ranging from +/-6 giving 1728 possible values, with the function ranging from 1 to 150 million. The probability of a random number intersecting the value of the mass ratios can be estimated from the ratio of the residual width, to the probability of a value in that range. Individually those probabilities are .0027, 0.15, 0.10, 0.13, 0.75, 0.15.3. The total probability of intersecting all six of the most experimentally accurately known to this accuracy is about one chance in 2 million.

The other issue is the residuals, which will be shown to be mathematically related to the Rydberg constant, which is not possible if the calculated values of the mass is not causally related to the actual mass.

This paper deals with an alternate approach to calculating mass ratios. It is well known that complex particles have internal multimode oscillations and a complex structure.

For example the leptons, though considered "elementary" must be more complex than just a single electron, and have often been considered as higher energy versions of the electrons. It will be shown that all complex particles Leptons, Bosons, & Mesons, have a similar multimode structure, and that the characteristic differences of the mass of the particles are in the values of the modes. The concept is not unlike developed by Dirac in establishing a radial mode of oscillation for the Muon [2], and in fact a radial mode is part of the makeup of the Muon developed here.

The residual values between the modes calculated values and the experimental values have been determined to have a quantum valued structure that appears to be related to an internal fine structure. This effect is small but there is a mathematical relation between the residuals of all the particles, eliminating the possibility that the mode calculations are random numbers.

## Single Mode

Starting with the Systemfunction for a single mode particle (electron) from Eq.24 [1] we have:

$$\tilde{\Theta} = Ae^{\left.(iS)^2\right|_{r_n=0}} \tag{1}$$

Ignoring the interaction of other particles, this can be evaluated at a single particle, and when neglecting interaction effects this becomes:

$$\tilde{\Theta} = \mathbf{e}^{\left[\pm \not{\eta}\, im \Re \not{\phi} \pm 1/2\right]^2}, \tag{2}$$

The ½ in this expression is the value of the spin projection along the z axis. This was adequate to describe the dynamic particle properties, but for internal structure the spin must be defined in terms of the Pauli spin vector matrix:

$$\boldsymbol{\sigma} = \sigma_1 + \sigma_2 + \sigma_3 \tag{3}$$

With:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{4}$$

Thus Eq.(2), is restated in more detail as:

$$\tilde{\Theta} = \mathbf{e}^{\left[-\not{\eta}^2\left(m\Re\not{v}\right)^2 \pm \not{\eta}\, imct\, \not{v}\boldsymbol{\sigma} \quad +1/4\,\boldsymbol{\sigma}^2\right]} \tag{5}$$

Note that the first term in the exponent is zero the second term is imaginary, and last terms is real, and contributes to the scale factor of the expression.

The radius of the system $\Re$ becomes just the local value $\Re = cT \rightarrow ct$ for the imaginary terms. The mass $m = mc/\hbar$, is the reciprocal of the Compton radius, and the velocity is in Feynman slash notation (For general conventions see appendix I). From Eq.(5), the deBroglie and Compton frequencies are:

$$\omega_d \;=\; mc\left|\vec{v}\,\right| \;\;\&\,,\;\; \omega_C \;= mc \tag{6}$$

The focus of this paper is particle rest mass, and so the velocity, and deBroglie frequency will be set to zero. Simplifying this is then:

$$\tilde{\Theta} = \mathbf{e}^{\left[\pm\, i\, mct\sigma + 1/4\sigma^2\right]} \tag{7}$$

.

**Spin Matrix Representation**

Appendix II discusses a multimode system and the coupling that results in normal modes in a mechanical model, but for a proper mathematical representation, the modes need to be defined in terms of the Pauli spin matrix. For the leptons it is expected that the modes described above can be properly incorporated into Eq. (7) by associating each modes with each of the orthogonal Pauli matrix.

Noting that the ½ term in Eq. (7)is actually the ½$\hbar$spin angular momentum projection along the direction of motion, and can be represented by (½$\sigma_3$). For the electron we normally assert the projections along each of the other axis also have an amplitude of 1/2 It is proposed that this is not necessarily the case, but that the amplitudes of the other axis can have values associated with integral wavelengths as suggested in the normal modes.(see appendix II) It is asserted that the other modes which are symmetric along the x and y axis can be assigned as matrix amplitudes. The spin matrix for a multimode particle with half spin along the z axis would be:

$$\sigma_M = \sqrt{j\pi}\,\sigma_1 + i\sqrt{k/\pi}\,\sigma_2 + \frac{1}{2}\sigma_3 \tag{8}$$

and the square, and real exponent is::

$$\boldsymbol{\sigma}^2 = j\pi + k/\pi + 1/4 \tag{9}$$

Putting Eq.(8), into Eq. (7) gives:

$$\tilde{\Theta} = \mathbf{e}^{\left[\pm i\,mct\sigma + j\pi + k/\pi + 1/4\right]} \tag{10}$$

The extra modes should not be interpreted as an angular momentum since the values don't correspond to known values for particles. From the following, it more likely that it represents the amplitude of standing waves symmetric with these axes.

It is asserted that the fundamental reference mass is the electron, and thus the contributions of the modes to the scale factor as noted in Eq.(33), would be such that the ratio of a particles mass to the mass of an electron is:

$$m/m_e = \left[\pm\, e^{\boldsymbol{\sigma}_A^2} + e^{\boldsymbol{\sigma}_B^2}\right] \tag{11}$$

A sum is expected since normal modes are in general a linear combination of different terms.

After some trial and error, values of j and k can be arrived at that give mass values very close to the actual mass values of many particles. The sequence of the j, k for the particles has patterns that are not random numbers, but indicate some underlying, yet to be determined, selection rules.

## Calculated Particle Mass Ratio

The values of j and k that give the proper values for the mass of the leptons in terms of electron masses will be:

$$\boldsymbol{\sigma}_A^2 = L\pi + M/\pi + S_A^2 \tag{12}$$

$$\boldsymbol{\sigma}_B^2 = J\pi + K/\pi + S_B^2 \tag{13}$$

Where j & k terms are designated J, K, L, & M , and , $S^2$ iz the z spin component.

## Leptons

Interpreting the terms in Eq.(11), to be two coupled coexisting parts of a particle, and for leptons that have ½ spin, the value of one of the S terms is zero and the other must be $S^2 = 1/4$ , The modes for the leptons are found to be:

Muon

Mode amplitude square
$$\boldsymbol{\sigma}_A^2 = 0\pi - 2/\pi + 1/4$$
$$\boldsymbol{\sigma}_B^2 = 2\pi - 3/\pi + 0 \tag{14}$$

Mass ratio

| Experimental | Calculated | Error in parts |
|---|---|---|
| 206.7682648 | 206.7575378 | 1/1850. |

Note that for the Muon as well as all the other particles, there are only three selectable values in the function and they range from +/- 1 to 6

Tauon

Mode amplitude square
$$\boldsymbol{\sigma}_A^2 = 0\pi + 6/\pi + 1/4$$
$$\boldsymbol{\sigma}_B^2 = 3\pi - 4/\pi + 0 \tag{15}$$

Mass ratio

| Experimental | Calculated | Error in parts |
|---|---|---|
| 3477.1502 | 3477.381104 | none |

## Quark Matrix Representation

By noting the mathematical relations between the Pauli matrix and the ud quarks the above procedure can be extended to at least the primary hadrons.

The composition of the hadrons are defined in terms of the SU(3) quark, or Gell-Mann matrix which are generalizations of the Pauli spin matrix, and the first three are identical to the Pauli matrix.

Using the quark wavefunctions noted by Xiangdong, [4] the meson wave function in terms of quarks for SU(2), is by noting the isosinglet I = 0 combination, is $q1\bar{q} \square \left(u\bar{u}+d\bar{d}\right)/\sqrt{2}$, where $\bar{q}=\left(\bar{u},\bar{d}\right)$. The isotriplet I = 1 is $\phi^i \square q\,\tau^i\,\bar{q}$ with i = 1, 2, 3, where $\tau^i$ are 3 Pauli matrices,

$$\phi^1=\left(u\bar{d}+d\bar{u}\right)/\sqrt{2}$$
$$\phi^2=i\left(d\bar{u}-u\bar{d}\right)/\sqrt{2}$$
$$\phi^3=i\left(u\bar{u}-d\bar{d}\right)/\sqrt{2}$$

Inserting these into Eq.(10), for the Pauli matrix allows:

$$\tilde{\Theta}= \mathbf{e}^{\left[\pm \not{\eta} i m \Re\ +\sqrt{j\pi}\left(u\bar{d}+d\bar{u}\right)+i\sqrt{k/\pi}\left(d\bar{u}-u\bar{d}\right)+\frac{1}{2}\left(u\bar{u}-d\bar{d}\right)\right]^2} \quad (16)$$

This displays the function as having meson type modes with integral wave orbital amplitudes. For L=0 particles, like the Proton and neutron, the waves must have zero orbital angular momentum, and thus be standing waves with integral wavelengths around the x and y axis.

Very accurate values of the mass ratios for the Proton and Neutron can be calculated, by picking certain values of the j, and k integers.

Proton amplitude square
$$\boldsymbol{\sigma}_A^2 = 0\pi - 1/\pi + 1/4$$
$$\boldsymbol{\sigma}_B^2 = 3\pi - 6/\pi + 0 \quad (17)$$

Neutron amplitude square
$$\boldsymbol{\sigma}_A^2 = 0\pi + 3/\pi + 1/4$$
$$\boldsymbol{\sigma}_B^2 = 3\pi - 6/\pi + 0 \quad (18)$$

Pi [0] Meson amplitude square [1]
$$(-)\quad \boldsymbol{\sigma}_A^2 = 0\pi - 3/\pi + 1/4$$
$$\boldsymbol{\sigma}_B^2 = 2\pi - 3/\pi + 1/4 \quad (19)$$

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[1] The pi 0 is the only presented particles with a negative sign for the first term in EQ.(11) .

Pi $^{\pm}$ Meson amplitude square

$$\boldsymbol{\sigma}_A^2 = 0\pi + 6/\pi + 1/4$$
$$\boldsymbol{\sigma}_B^2 = 2\pi - 3/\pi + 1/4 \tag{20}$$

Note that the Pi mesons have a spin of zero with two opposite components. The square of each component is ¼, and both contribute to the total mass.

The following is a table listing the values and giving the error in one part per the experimental value. Most of the calculated values are well beyond a probability of random coincidence.

Putting the mode values Eq.(14), Eq.(15), Eq.(17), Eq.(18), Eq.(19), and, Eq.(20), into Eq.(11), gives the values shown in table 1. All masses are expressed in units of electron masses, and the residuals are the difference between the experimental, and mode calculated values.

**Table 1.** *Modes and Particle mass ratios*

| Particle | Modes | Experimental Values | Mode Calc. Values | Residuals |
|---|---|---|---|---|
| Proton | $f_1(0,-1,\ 1/4)+f_2(3,-6,\ 0)$ | 836.15267245(75) | 1836.15213467 | 0.00053777(75) |
| Muon | $f_1(0,-2,\ 1/4)+f_2(2,-3,\ 0)$ | 206.7682913(68) | 206.757537039 | 0.0107535(35) |
| Tauon | $f_1(0,\ 6,\ 1/4)+f_2(3,-4,\ 0)$ | 3477.15(31) | 3477.381104 | 0 |
| Neutron | $f_1(0,\ 3,\ 1/4)+f_2(3,-6,\ 0)$ | 1838.683722(41) | 1838.55468750 | 0.129034(41) |
| Pion± | $f_1(0,\ 6,\ 1/4)+f_2(2,-3,\ 1/4)$ | 273.13205(68) | 273.279548677 | - 0.14749(68) |
| Pion0 | $f_1(0,-3,\ 1/4)+f_2(2,-3,\ 1/4)$ | 264.1421(7) | 264.115487482 | 0.0266(7) |

Note that all the L's are zero, and the pions, have spin included in both the $f_1$, and $f_2$ term.

Simple calculations show that the probability of this series randomly yielding the proper mass ratio values to this accuracy for the six particles to be less one in two million. There is undoubtedly a causal relation.

## Residuals

The most important point is that the values of the residuals are not random numbers, but are all integral values of a single Rydberg constant. This fact precludes the possibility that the mode calculated values are random numbers and not causally related, for the simple fact that random numbers, only close to the mass values would yield random residuals.

The residuals are relatively small, and most notable is that the calculated value of the Proton is within one part in three million of the actual mass value. Because of the high accuracy of the experimental value of the Proton mass, the residual is known to a high accuracy. In addition the experimental accuracy of five other particles is known with sufficient accuracy to compare evaluate relations among the residuals of these particles. It will be shown that the residuals are quantized and mathematically related.

The selected matched particles have high experimental accuracy, and represent the simpler particles, notably the leptons and "ud" quark particles. Many other baryons and mesons matches can be found, but the experimental error bars are too wide to make use of the data.

The general expression for a spin ½ particle mass as illustrated above is:

$$m = m_e \left[ \pm e^{\left(I\pi - K/\pi + 1/4\right)} + e^{\left(J\pi - L/\pi + 0\right)} \right] \qquad (21)$$

Presuming that the modes represent particle circulation in the nucleus then each mode has a path, and each path would be modified by the quantum effects.

$$j\pi + f_J(\alpha) \qquad \frac{k}{\pi} + f_k(\alpha) \qquad (22)$$

Noting that if the f functions are small:

$$e^{j\pi + f_J(\alpha) + \frac{k}{\pi} + f_k(\alpha)} = e^{j\pi + \frac{k}{\pi}} + f_j(\alpha) + f_k(\alpha) \qquad (23)$$

Thus:

$$m = m + f_j(\alpha) + f_k(\alpha), \qquad (24)$$

this makes the residuals the simple sum of the included effect.

First to note is the relative small size of the residuals. The mass is all expressed in electron masses, but the range is between, .25 and 100 kev.comparied to the marticles that are in hundreds of Mev.

Most notable is that each of the residuals, accept the $\mathrm{Pion}^{\pm}$, can be shown to be a product of a constant and a series of integral numbers. The operative constant is the Rydberg energy constant.

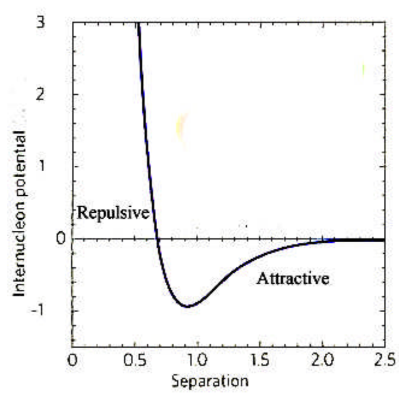
The Rydberg constant expressed in electron mass units is:

$$R_{\infty} = \frac{\alpha^2}{2} \qquad (25)$$

Noting that the smallest residual, that being the Proton is 0.00053778(75), It is pointed out that this residual is nearly exactly 20 times the Rydberg constant which in electron masses is 0.000532514.

The small differential (0.9%) can be attributed to the per nucleon repulsive mass increase for particles residing in the nucleus. As is well known, the energy potential for particles in the nucleus is repulsive, and the per-nucleon potential is about 8.8 Mev (Fig.1). The proportional mass increase of a proton with 8.8 Mev., of repulsion. Is: 8.8 /938.2,

**Figure 1**




If that proportion is applied to the Rydberg energy the modification is about (0.9) percent, and would be attributed to an energy level change for a particle in a repulsive potential residing in the core of the nucleus.

$$R_M = R_\infty\left(1+\frac{8.8}{938.2}\right) = R_\infty(1.009378943) \quad (26)$$

With this slight modification the residuals can be accurately expressed as:

$$20\times R_M = 0.000537507978 \quad (27)$$

Experimental = 0.00053777(75)

Multiplying it again by 20 gives the residual for the Muon

$$20\times 20\times R_M = 0.010750159564 \quad (28)$$

Experimental = 0.0107535(35)

Multiplying it again by 12 gives the residual for the Neutron:

$$12\times 20\times 20\times R_M = 0.129001914765 \quad (29)$$

Experimental = 0.129034(41)

Multiplying the Proton residual EQ.(27), by 50 gives the residual for the $\text{Pion}^0$:

$$50\times 20\times R_M = 0.026875398909 \quad (30)$$

Experimental = 0.0266(7)

Multiplying the Proton residual by $2/\alpha$ gives the $\text{pion}^\pm$ residual:

$$\frac{2}{\alpha}\times 20\times R_M = 0.147315886286 \quad (31)$$

Experimental = 0.14749(68)

Note that the calculated values are within the error bars, and each of these residuals are exact integral multiples of the same Rydberg constant. For the first three, a change in the value of the constant, or the integers by more than a few thousandths will cause the values to fall outside the error bars. The pions which do not have as accurate experimental mass have a little leeway in the multipliers. Both the 50, in the $\text{Pion}^0$, and the $2/\alpha$ in the $\text{Pion}^\pm$ could be $\pm 1$, and still be within the experimental error bars.

## Shell Model

The notable feature that all the residual multipliers above, are nuclear Shell Model magic numbers:

2, 8, 12, 20, 28, 50, 82, 126

These numbers are generally the result of the filled shells of a three dimensional oscillator with spin degeneracy. Looking specifically at the proton, 20 is the magic number for the first three filled shells of 2, 6, and 12.

level 0: 2 states ($l$ = 0) = 2.
level 1: 6 states ($l$ = 1) = 6.
level 2: 2 states ($l$ = 0) + 10 states ($l$ = 2) = 12.
level 3: 6 states ($l$ = 1) + 14 states ($l$ = 3) = 20

In addition, the modes for the Proton, 1. 3. 6 which are the j and k coefficients of quarks:

$$j\pi\left(u\bar{d}+d\bar{u}\right)^2 - k/\pi\left(d\bar{u}-u\bar{d}\right)^2 \qquad (32)$$

This suggests that each of the modes represents the levels for two states $u\bar{d}$, and $d\bar{u}$. Thus there would be two states per level and the quark pairs in the levels would be 2, 6 ,12, thus giving the magic number of 20 quark pairs.

All the other residuals are products of the proton residual, and are also magic numbers. The presumption wound be that the others should be looked at in terms of the multiple permutations of entangled shell systems. The Muon residual is 20 times the proton residual implying the interaction of two, 3 level systems (20x20). The neutron would then be the interaction of three systems (20x20x12), two with the first three levels filled, and one with only 4 pairs in the third level.

The multiplier of $2/\alpha$ for the $\text{Pion}^{\pm}$ suggests a different mechanism than the entangled nuclear structures of the other particles. The sign for this residual is negative, implying an attractive potential, and the magnitude would be that for a single charge at the distance of the electron radius from the central nuclear shell.

Adding the residuals to the above mode calculated values, puts the calculated value of the mass ratio, within the current experimental error bars.

**Table 4.** *Summary with calculated residuals*

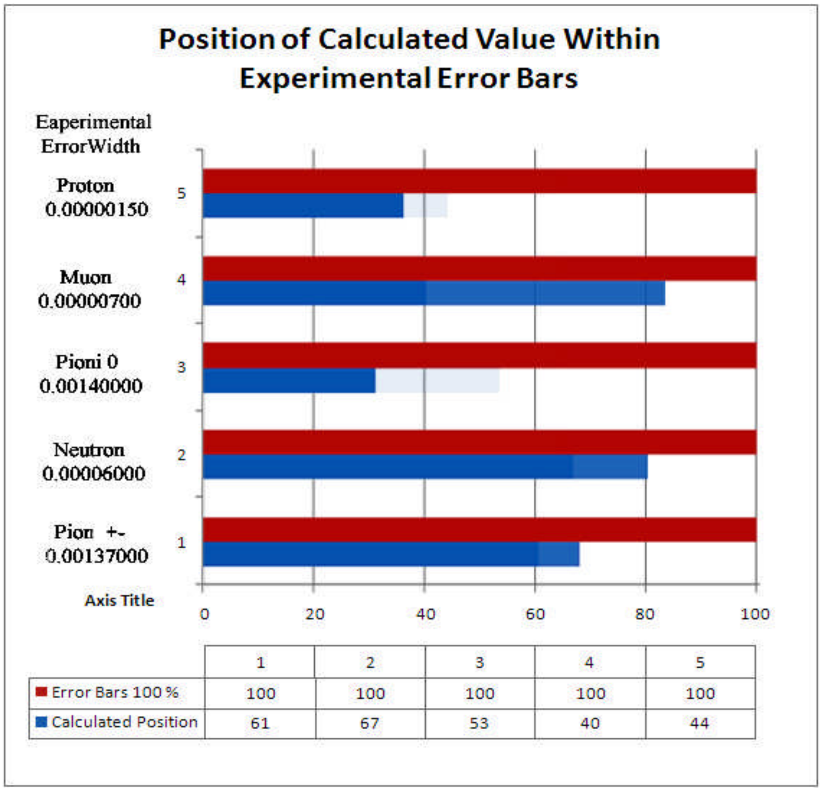
| | Experimental | Exp. Accuracy |
|---|---|---|

| | | | |
|---|---|---|---|
| Proton | | 1836.15267245(75) | |
| | Calculated | 1836.15267217 | 1/ 2.44E+09 |
| Muon | | 206.7682913(68) | |
| | Calculated | 206.7682871 | 1/ 2.95E+07 |
| Neutron | | 1838.683722(41) | |
| | Calculated | 1838.683689 | 1/ 3.06E+07 |
| Pion+ | | 273.13205(68) | |
| | Calculated | 273.13223 | 1/ 1.99E+05 |
| Pion$^0$ | | 264.1421(7) | |
| | Calculated | 264.1423 | 1/ 1.89E+05 |

The mathematical relationship between these residuals, removes any question that the mode calculations are the result of random numbers.

The position of the calculated value can be placed in the experimental error bars as noted by the Particle Data Group [3]. The chart (Graph 1), shows the calculated position within the experimental error bars.

**Graph1.** *Accuracy Chart*




CONCLUSION

This paper has presented a plausible, relatively straightforward explanation of particle mass ratios, and the specific values for some well known particles. The additional nuclear modes postulated, are similar to the Schrödinger integral modes for an atom. The matching values of the experimental mass ratios, and the mode calculated values has a probability of less than one in two million (five sigma) of being random. Including the residuals, the mass ratios for the Proton, Neutron, Muon, Tauon, and Pions, defines the mass values within the experimental margin of error. The reduction of the experimental errors in the particle masses, will be a further test the merit of this paper. QED.

Références:

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**[3]** J. Beringer et al. (Particle Data Group), J. Phys. D**86**, 010001 (2012).

**[4]** Hitoshi Murayama Lecture Notes http://hitoshi.berkeley.edu/129A/strong1.pdf

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# Appendix I

## *Definitions and Conventions*

The radius of particle system $\Re = cT = \Re_0 + ct$

Four velocity $\gamma^\mu \left(\frac{v}{c}\right)_\mu = \not{v}$ unitless

Null unit vector $\not{\eta} = {}^\mu\gamma\eta_\mu = \left(\gamma^0 + \vec{\eta}\cdot\vec{\gamma}\right)$ $\vec{\eta}\cdot\vec{\eta} = -1$

Mass in this paper $m = \frac{mc}{\hbar} = \frac{1}{\lambdabar}$

Rest mass $m_0 = \frac{m_0 c}{\hbar}$

Compton radius $\lambdabar = \frac{\hbar}{mc}$

The Dirac gamma matrix convention:

$$\gamma^1 = \begin{bmatrix} & & & +1 \\ & & +1 & \\ & -1 & & \\ -1 & & & \end{bmatrix} \quad \gamma^2 = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix} \quad \gamma^3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix} \quad \gamma^0 = \begin{bmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{bmatrix}$$

$$\gamma^1\gamma^1 = -1, \quad \gamma^2\gamma^2 = -1, \quad \gamma^3\gamma^3 = -1, \quad \gamma^0\gamma^0 = +1.$$

Pauli spin matrix:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Feynman slash notation:

$$\not{A} = \gamma^\mu A_\mu \qquad \not{a}\,\not{a} = a^\mu a_\mu = a^2 \qquad \not{a}\,\not{b} + \not{b}\,\not{a} = 2\,\not{a}\cdot\not{b}$$

$$\not{v} = \left(\gamma^1 v_x + \gamma^2 v_y + \gamma^3 v_z + \gamma^0 c\right) / c$$

## Appendix II

### Normal Modes General Concept (section for concept only)

(This development was the original motivation for evaluating the unusual correlation of the hydrogen type modes with the mass ratios)

From analogy with mechanical systems with normal coordinates, in general the solution of a complex coupled system is a sum of the different solutions of the various modes, thus if a particle is a composition of coupled normal modes:

$$\tilde{\Theta} = a_1 e^{f_1(\omega)} + a_2 e^{f_2(\omega)} \tag{33}$$

with $a_1$ and, $a_2$ being scale factors proportional to the particle masses.

From analogy, also with coupled mechanical systems, we will propose for the internal coupling of a multimode particle the relation among the frequencies such that:

$$\omega = \omega_C \pm 2\sqrt{\omega_C\omega_1 - \omega_C\omega_2} \tag{34}$$

A review of normal mode coupling in mechanical systems illustrates that this is not an unreasonable form.

It is easy to determine the connecting relations between the frequencies for this coupling from the coupling matrix. These frequencies have to be eigenvalues, thus.

$$\begin{bmatrix} \omega & & \\ & \omega_C + 2\sqrt{\omega_C\omega_1 \pm \omega_C\omega_2} & \\ & & \omega_C - 2\sqrt{\omega_C\omega_1 \pm \omega_C\omega_2} \end{bmatrix}, \tag{35}$$

or

$$0 = \omega_C\left(\omega_C + 2\sqrt{\omega_C\omega_1 - \omega_C\omega_2}\right)\left(\omega_C - 2\sqrt{\omega_C\omega_1 - \omega_C\omega_2}\right), \tag{36}$$

leading to the requirement that the sum of the frequencies be constant.

$$\left(\omega_C - 4\omega_1 \pm 4\omega_2\right) = K \tag{37}$$

Presuming frequency and energy are related, then the proposed coupling is just the condition that the sum of the energy be a constant.

$$E = \left(\omega_C - 4\omega_1 \pm 4\omega_2\right)\hbar \tag{38}$$

$\omega_C = mc$ is the Compton frequency and $\omega_1$ and $\omega_2$ are the frequencies of the other two modes.

Factoring and presuming a constant ratio of the frequencies gives:

$$\omega = \omega_C\left(1 \pm 2\sqrt{\left(a_1 - a_2\right)}\right), \tag{39}$$

There has to be a fixed non-temporal ratio of the frequency, otherwise mass ratios would not be constant. It is expected that any oscillation, within a particle nucleus has to have an integral number of wavelengths related to the Compton cycle to exist as a stable particle. The Compton cycle will be regarded as a single azimuthal cycle.

The primary Compton mode for the simple particle i.e. the electron is proposed to be a circumferential azimuthal wave with the effective radius of $\bar{\lambda} = \hbar / mc$ and having a circumference of $\lambda = 2\pi\bar{\lambda}$ . Analogously with an atomic system two other modes of oscillation can be defined in similarity with the radial, and colatitude modes. The first as a spherical radial wave, having an integral number of wavelengths within the Compton radius, thus:

$$\frac{\bar{\lambda}_C}{\lambda_R} = j \quad , \tag{40}$$

with frequencies:

$$\frac{\omega_R}{\omega_C} = \frac{\lambda_C}{\lambda_R} = j\pi \tag{41}$$

The third mode, is a colatitude oscillation having an effective integral wavelength at a distance from the center of mass of $r = \lambda_C / 2k$ where k is an integer, thus:

$$\frac{\omega_L}{\omega_C} = \frac{\lambda_C}{\lambda_L} = k / \pi \qquad (42)$$

The radial position and wavelengths of the modes is analogous to the Schrodinger modes for which proper treatments would be the path integral formulation. This choice of the colatitude radius is empirical, but will fit the observed mass values.

With these defined modes, Eq. (39) , becomes:

$$\omega = \omega_C \left(1 \pm 2\sqrt{j\pi - k / \pi}\right) \qquad (43)$$

This "general concept" development is intuitive approach to understand the physical results. Replacing the $\sigma$ in Eq. (7) with $\left(1 \pm 2\sqrt{j\pi - k / \pi}\right)$ will give the same results as the development in the following section, but lacks the proper mathematical apparatus.