Particle Mass Ratios

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V090210

Abstract

Based on the developments in a previous paper, this paper presents straightforward explanation of particle mass ratios, and the specific values for some well known particles. The additional nuclear modes postulated, are similar to the Schrödinger modes in the atom, and, though speculative, the mass ratios calculated for elementary particles are very close the observed mass ratios.

Introduction

This section deals with an interesting approach to calculating mass ratios, and although reasonable, the actual details are **quite speculative**.
In a previous paper "A Multiple Particle System Equation" [1] a single mode, half spin, matrix particle function was developed that has the characteristics of an electron / positron. More complex particles have multimode oscillations and a more complex structure. Even higher leptons, though considered "elementary" must be more complex than just a single electron. This paper will focus on defining modes of oscillation that geometrically couple with the Compton oscillation, such that there are an integral number of cycles completed in the same time frame as the Compton oscillation, and contribute to the rest mass of the particle. This is not unlike the concept developed by Dirac in establishing a radial mode of oscillation for the Muon [2], and in fact a radial mode is part of the makeup of the Muon developed here.

We also expect that the Clifford-Dirac SU(2) matrix representations of the systemfunctions developed in [1], are inadequate for the more complex structures, and must be enhanced by the direct products of the Gell-Mann matrices i.e. $SU(2)\otimes SU(3)$. There has to be a relation between the proposed oscillatory modes, and the quark content, but at this point, that has not been established.

Single Mode

From [1] we have:

$$\vec{\psi}_n = \mathbf{exp}\left(\pm i \frac{\Re}{\mathbf{r}_n}\frac{\vec{V}_n}{c} \pm \frac{1}{2}\right)^2, \tag{1}$$

or

$$\vec{\psi}_n = \mathbf{exp}\left[-\left(\frac{\Re}{\mathbf{r}_n}\frac{\vec{V}_n}{c}\right)^2 \pm i \frac{\Re}{\mathbf{r}_n}\frac{\vec{V}_n}{c} + \frac{1}{4}\right] \tag{2}$$

Where $\Re \rightarrow ct$ for the imaginary terms, and $\mathbf{r}$ is the particle Compton radius $\mathbf{r} = \hbar / mc$. (For general conventions see appendix II). From this he deBroglie and Compton frequencies are:

$$\omega_d = \frac{M_{0n}cv}{\hbar} \quad \&, \ \omega_C \quad \frac{M_{0n}c^2}{\hbar} \tag{3}$$

Simplifying by letting the velocity be zero we have:

$$\vec{\psi} = \mathbf{e}^{\left[-\left(\frac{\Re}{\mathbf{r}_n}\frac{\vec{V}}{c}\right)^2 \pm i\gamma_4\omega_c + \frac{1}{4}\right]} \tag{4}$$

Where ω_c is the Compton frequency:

From analogy with mechanical systems, with normal coordinates we know, in general, that the solution is a sum of the different independent modes of oscillation, thus in general:

$$\vec{\psi} = k_1 e^{f(\omega_1)} + k_2 e^{f(\omega_2)} \tag{5}$$

Where k_1 and, k_2 are in general complex scale factors.
From analogy also with coupled mechanical systems we will propose for a multimode solution, coupling, such that:

$$\omega = \omega_C \pm 2\sqrt{\omega_C\omega_1 - \omega_C\omega_2} \tag{6}$$

Where $\omega_C = Mc^2/\hbar$ is the Compton frequency.

The two modes we will propose are coupled to the Compton frequency such that

$$\omega = \omega_C \left(1 \pm 2\sqrt{\frac{\omega_1}{\omega_C} - \frac{\omega_2}{\omega_C}}\right) = \omega_C \left(1 \pm 2\sqrt{(a_1 - a_2)}\right), \tag{7}$$

where there has to be a fixed non-temporal ratio of the frequency, otherwise mass ratios would not be constant. It is expected that any oscillation cycle within a particle nucleus has to be completed within an integral number of cycles of the Compton cycle, or vice versa, else the particle could not exist as a particle.

Multiple Modes

We will presume that the normal Compton mode is a circular wave with the radius of $\mathbf{r} = \hbar/mc$ and having a circumference of $\lambda = 2\pi\mathbf{r}$. We will define two other particular modes of oscillation in similarity with the radial, and azimuthal modes of the Schrödinger atom. The first as a spherical radial wave, having an integral number of wavelengths across the diameter of the Compton radius. Thus:

$$\frac{r_C}{\lambda_1} = n \tag{8}$$

and,

$$a_1 = \frac{\omega_1}{\omega_C} = \frac{\lambda_C}{\lambda_1} = n\pi \tag{9}$$

And the second mode as an azimuthal mode at a decreasing radius of $r = \lambda_C/2m$ thus:

$$a_2 = \frac{\omega_2}{\omega_C} = \frac{\lambda_C}{\lambda_2} = m/\pi \tag{10}$$

Note that this choice of the azimuthal radius is quite arbitrary, and picked to fit the observed mass values.

Eq. (7) , then becomes:

$$\omega = \omega_C\left(1 \pm 2\sqrt{n\pi - m/\pi}\right) \tag{11}$$

Attempting to insert this directly into Eq. (1) , and Eq. (4) , in its current form results in non-physical terms that improperly represent reality, thus the necessity for evoking the direct products of the SU(2)or SU(3) representation for the wavefunctions. This should not be surprising since SU(3) is the fundamental generators representation the Quark constituents of complex particles. We should note that the model being proposed is more of a physical model of the particles, than a probability representation, and although there is a connection the functional representations are not the same [1].

For the purposes here, we need only use three of the Gell-Mann matrix, which are the same as two of the Pauli matrix along with the unit matrix. Note that the Gell-Mann matrix are generalizations of the Pauli matrix, and in two dimensions are the same[4].
Comparing with Eq. (4), we can write:

$$\vec{\psi} = \mathbf{e}^{\left(i\frac{\Re\,\vec{V}_n}{\mathbf{r}_n\,c} \pm \frac{1}{2}\sigma_1 + \sqrt{n\pi - m/\pi}\,\sigma_2\right)^2} , \tag{12}$$

and:

$$\vec{\psi} = \mathbf{e}^{\left[-\left(\frac{\Re\,\vec{V}_n}{\mathbf{r}_n\,c}\right)^2 \pm i\left(\frac{\Re\,\vec{V}_n}{\mathbf{r}_n\,c}\right)\otimes\left[\sigma_1 + 2\sigma_2\sqrt{n\pi - m/\pi}\right] + \frac{1}{4} + \left(n\pi - m/\pi\right)\right]} \tag{13}$$

Comparing with Eq.(2) , and Eq.(4) , we have two extra terms.

$$\frac{Mc^2t}{\hbar}\left[\sigma_1 + 2\sigma_2\sqrt{n\pi - m/\pi}\right] = \frac{Mc^2t}{\hbar}\begin{bmatrix} 0 & 1 - i\sqrt{n\pi - m/\pi} \\ 1 + i\sqrt{n\pi - m/\pi} & 0 \end{bmatrix} \tag{14}$$

and:

$$+\left(n\pi - m/\pi\right) \tag{15}$$

The first are normal multimode frequencies, and the second is a real term that adds into the scale factors of Eq.(5) .

Comparing Eq.(4) , and Eq.(5), the single mode or electron wavefunction can be factored out, giving:

$$\vec{\psi} = \left[e^{\left(n_1\pi - m_1/\pi\right)} + e^{\left(n_2\pi - m_2/\pi\right)}\right]e^{f(\omega_e)} \tag{16}$$

after considerable trial and error, values of m and n can be arrived at that give mass values very close to the actual mass values of common well known particles. The sequence of the m, n for the particles have patterns that do not appear to be random numbers, but indicate some underlying selection rules.

Calculated Particle Mass Ratio

Table 1

Particle	Modes	Calc.Value	Actual Mass Ratio	Error
Electron	$e^{0}\left(1\right)$	1		
Muon	$e^{2\pi-\frac{3}{\pi}}\left(1+e^{-2\pi+\frac{2}{\pi}}\right)$	206.805565	206.7682662	1/ 5500
Tauon	$e^{3\pi-\frac{4}{\pi}}\left(1+e^{-2\pi+\frac{1}{\pi}}\right)$	3477.61673	3477.561 ±.58	0
Neutron	$e^{3\pi-\frac{6}{\pi}}\left(1+e^{-2\pi+\frac{0}{\pi}}\right)$	1838.645328	1838.68366	1/ -48000
Proton	$e^{3\pi-\frac{6}{\pi}}\left(1+e^{-3\pi+\frac{6}{\pi}}\right)$	1836.218164	1836.15268	1/ 28000

Note that the calculated values of the masses are extremely close, and there are no arbitrary constants. The least accurate is still within one part in 5500 of the experimental value, and the Tauon is within experimental error.. Although, at this time, there is not an established connection between the n, m modes, and the quark content of the nucleon, there is undoubtedly some connection.

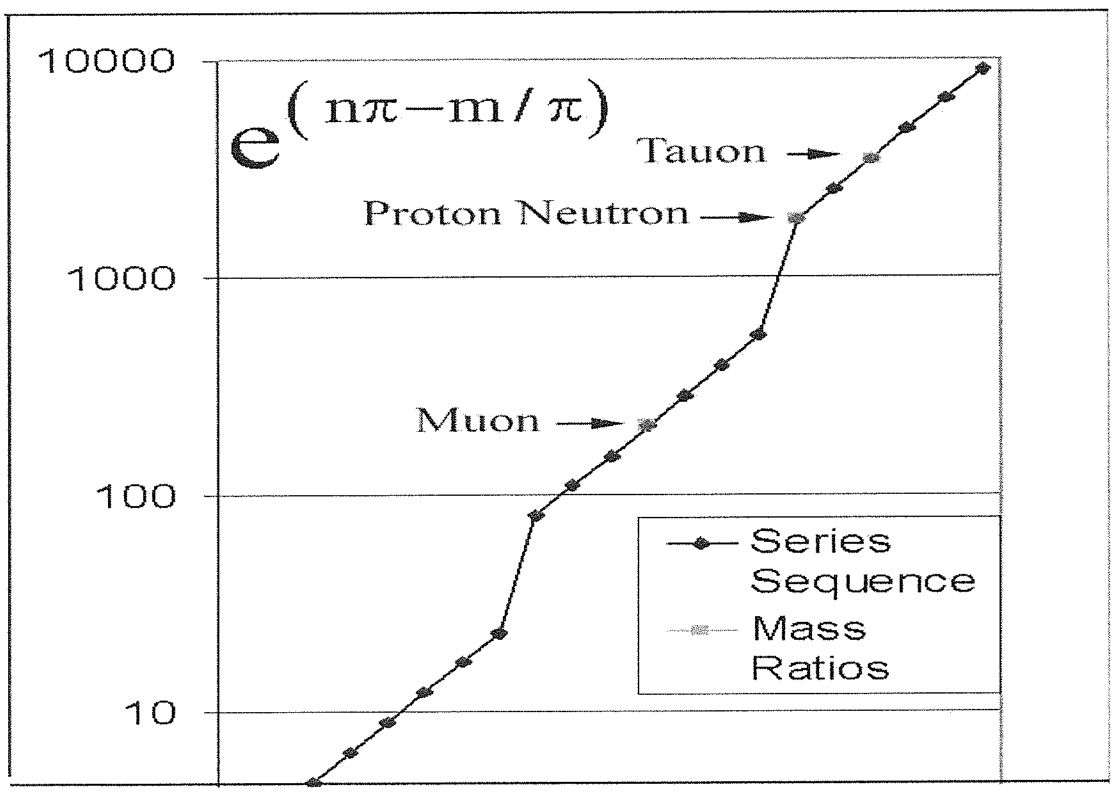
An alternate factoring of the particle ratios in Appendix II shows more clearly, the non-randomness of the sequence.

Random Coincidence

The first question is: Is the series of the n, m numbers, capable of generating "any" random number, or are the mass ratio values generated by this series unique?

The series can generate a large number of values, and if the values of n and m are large enough. One can get closer and closer to any number. With the value of m and n restricted to numbers 0 through 6 there are only 49 possible values. And the fact that the first term of these four values hit within less than half a percent has a probability of about 1 in 40 thousand. The possible levels of the sequence can be displayed, and it can be noted that the sequence step levels of the series are 37% or greater, whereas the ratio levels are less than ½ percent off these sequence numbers. Graph 1 shows the relative values.

Graph I



Mass levels for the Proton, Neutron, Tauon and Muon

Residuals

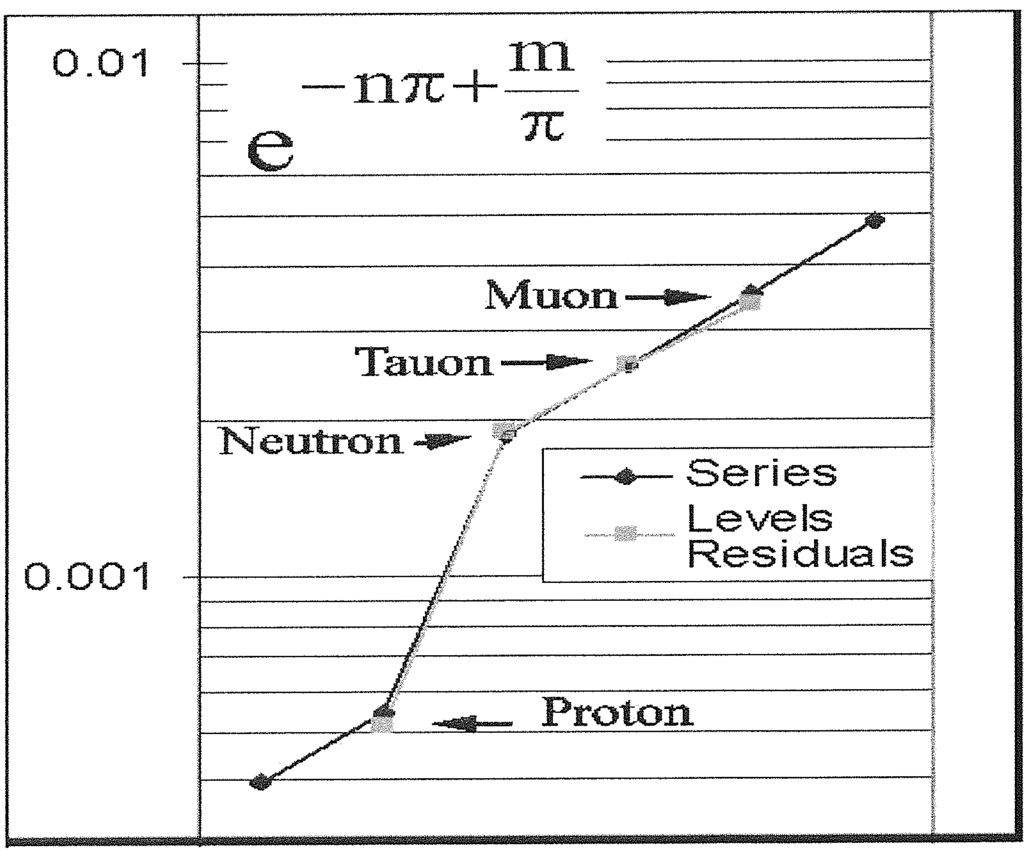
The second term of the particle mass ratios in table 1 are small, and if we look on them as residuals, and compare them with the actual residuals, we again come up with non-random numbers.

Table 2

Particle	Calculated Residual		Actual Residual	% Error
Muon	$e^{-2\pi+\frac{2}{\pi}}$	$= 0.0035296183$	0.003348625	5.4
Tauon	$e^{-2\pi+\frac{1}{\pi}}$	$= 0.0025673644$	$1.0027 \pm .0003$	0
Neutron	$e^{-2\pi+\frac{0}{\pi}}$	$= 0.0018674427$	0.001888329	1.1
Proton	$e^{-3\pi+\frac{6}{\pi}}$	$= 0.0005448943$	0.000509213	7.0

Again we can illustrate this in graphical form, showing the non-randomness of the residuals in relation to the sequence.

Graph II



Mass residuals for Proton, Neutron, Tauon, and Muon

Modes for other Particles

It is somewhat strange that the integral modes we have defined, fit both leptons, and baryons, particularly since the wavefunctions for the proton and neutron are linear sums of nine components, whereas the leptons are considered to be elementary particles.

Proton:

$$|p\rangle = \frac{1}{3\sqrt{2}} \begin{bmatrix} 2\left(|u\uparrow u\uparrow d\downarrow\rangle + |u\uparrow d\downarrow u\uparrow\rangle + |d\downarrow u\uparrow u\uparrow\rangle\right) \\ -\left(|u\uparrow u\downarrow d\uparrow\rangle + |u\uparrow d\uparrow u\downarrow\rangle + |d\uparrow u\uparrow u\downarrow\rangle\right) \\ +\left(|u\downarrow u\uparrow d\uparrow\rangle + |u\downarrow d\uparrow u\uparrow\rangle + |d\uparrow u\downarrow u\uparrow\rangle\right) \end{bmatrix} \tag{17}$$

In addition, with a much simpler wavefunction it would be expected that the mesons, particularly the π mesons, would fit into the series, however they do not seem to fit at all.

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \tag{18}$$

The matched mass levels fit both leptons, as well as baryons, thus the modes are not tied directly to the SU(3) group.

Though not as significant, and not inclusive of all Baryons, two others particles that do fit within experimental error are the baryon $\Delta(1620)$ resonance at 3170 electron masses:

$$e^{3\pi-\frac{6}{\pi}}\left(1+e^{-\frac{1}{\pi}}\right) = 3170.26 \text{ electron masses}, \tag{19}$$

and the strange baryon Ξ^0 $\left(1314{:}83 \ \pm 0{:}20 \text{ MeV}\right)$ at $2573.058 \pm .39$ electron masses

$$e^{3\pi-\frac{6}{\pi}}\left(1+e^{-2\pi+\frac{1}{\pi}}\left(e^{+\pi+\frac{6}{\pi}}+e^{-\pi+\frac{6}{\pi}}\right)\right) = 2572.7880 \tag{20}$$

Though the sequence fits the $\Delta(1620)$ very well $\Delta(1620)$, is actually a broad resonance, and the strange Ξ^0 (1314.83) is more exact, but an additional, though symmetric, term is required.

As in the case, when the pattern of energy levels of the Bohr atom were known, but not understood, there seems to exist the same circumstance. Until the dynamics of the mechanism are fully clarified this will remain somewhat of an enigma.

CONCLUSION

This paper has presented a relatively straightforward explanation of particle mass ratios, and the specific values for some well known particles.
The additional nuclear modes postulated, are similar to the Schrödinger modes in the atom, and, though speculative, the mass ratios calculated for elementary particles are very close the observed mass ratios. Even if the theoretical basis of the explanation is in error, the displayed sequences are too orderly to be random, and need explanation.

References :

[1] A Multiple Particle System Equation Underlying the Klein-Gordon-Dirac-Schrödinger Equations D.T. Froedge V071310 2010 http://www.arxdtf.org/

[2] P.A.M. Dirac, Proc. Roy. Soc. A268 (1962) 57.

[3] S. Eidelman et al. (Particle Data Group), Phys. Lett. B 592, 1 (2004) and 2005 partial update for edition 2006 (URL) **http://pdg.lbl.gov:**

[4] Hitoshi Murayama Lecture Notes http://hitoshi.berkeley.edu/129A/strong1.pdf

Appendix I

Definitions and Conventions

The radius of the universe:

$$\Re = cT = \Re_0 + ct$$

The Dirac matrix convention used in this development is

$$\gamma_1 = \begin{bmatrix} & & & -1 \\ & & -1 & \\ & +1 & & \\ +1 & & & \end{bmatrix} \quad \gamma_2 = \begin{bmatrix} & & & i \\ & & -i & \\ & -i & & \\ i & & & \end{bmatrix} \quad \gamma_3 = \begin{bmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{bmatrix} \quad \gamma_4 = \begin{bmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{bmatrix},$$

and

$$\gamma_1^{\,2} = -1, \quad \gamma_2^{\,2} = -1, \quad \gamma_3^{\,2} = -1, \quad \gamma_4^{\,2} = +1 .$$

$$\vec{S}\ \gamma_n = \gamma_2\gamma_3, \quad \gamma_1\gamma_3, \quad \gamma_1\gamma_2$$

Which are the elements of the spin vector:

$$\sigma_1 = \gamma_2\gamma_3 \qquad \sigma_2 = \gamma_3\gamma_1 \qquad \sigma_3 = \gamma_2\gamma_1$$

The vector four velocity:

$$\vec{V} = \gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c$$

Commutation relation with V and S

$$\begin{aligned}
\vec{S}\vec{V} &= \vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right) \\
\vec{V}\vec{S} &= \vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z + \gamma_4 c\right)\vec{S} \\
\vec{S}\vec{V} + \vec{V}\vec{S} &= 2\ \vec{S}\left(\gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z\right) \\
&= 2\ \left(\gamma_2\gamma_3 v_x + \gamma_1\gamma_3 v_y + \gamma_1\gamma_2 v_z\right) \\
&= 2\ \sigma \bullet V
\end{aligned}$$

Where $\sigma \bullet V$ is the Dirac spin vector, and V is the three velocity:

$$V = \gamma_1 v_x + \gamma_2 v_y + \gamma_3 v_z$$

Appendix II

An alternate factoring of the Particle Ratio function shows more clearly that the sequence is not a sequence of random numbers .

Particle	Electron Particle Mass Ratio
Muon	$e^{\pi-\frac{2}{\pi}}\left(e^{\left(\pi-\frac{1}{\pi}\right)} + e^{-\left(\pi-\frac{1}{\pi}\right)}\right)$
Tauon	$e^{2\pi-\frac{2}{\pi}}\left(e^{\left(\pi-\frac{2}{\pi}\right)} + e^{-\left(\pi+\frac{1}{\pi}\right)}\right)$
Neutron	$e^{2\pi-\frac{3}{\pi}}\left(e^{\left(\pi-\frac{3}{\pi}\right)} + e^{-\left(\pi+\frac{3}{\pi}\right)}\right)$
Pr oton	$e^{2\pi-\frac{3}{\pi}}\left(e^{\left(\pi-\frac{3}{\pi}\right)} + e^{-\left(2\pi-\frac{3}{\pi}\right)}\right)$