The Velocity of Light in a Locally Conserved Gravitational Field

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ABSTRACT

From earlier papers, on the theory of a locally conserved gravitational field, the relationship between gravitational potential and a gradient speed of light was developed, [1], [2], [3]. The concept of gravitation being simply a gradient in the velocity of light [4], has been proposed. In order to pursue the relationship between the index of refraction generated by gravitation and the index of refraction modified by the vacuum polarization effects of QFT, the value in flat space and flat time of the needs to be well defined. The index is well known to be asymmetric with respect to the radial vector, and can be deduced from existing experimental measurements.

Gravitationally Induced Index of Refraction

The tangential index of refraction of light induced by gravitating body can be deduced using the deflection of starlight and of Fermat’s principle of least action. Blandford & Thorne [5] have shown by projecting the photon trajectories of the Einstein metric onto flat space the index is:

\[ \eta = \left(1 - 2\frac{\mu}{r}\right)^{-1} \]  

(1)

Karimi, & Khorasani [6], have shown that with a more detailed development of the asymmetric aspects of the GR metric, that the index of refraction is actually:

\[ \eta = (1 + \phi)^{-1/2} \left(1 + \phi \cos^2 \theta\right)^{1/2} \quad , \quad \phi = \frac{r_s}{r} = \frac{2\mu}{r} \]  

(1.2)

The angle \( \theta \) is the angle between the wave vector and the radius. By dropping second order terms and simplifying, the velocity becomes:
\[ c = \left(1 - \left(1 + \cos^2 \psi \right) \frac{\mu}{r} \right) \] (1.3)

It is shown in Appendix II that this index of refraction gives the proper angle of deflection and Shapiro delay for a solar grazing photon.

This same result can be arrived at by using the result of Eq.(1), for the tangential motion, and noting for flat Minkowski geometry frequency must be constant the only change to a photon rising in a gradient in c is the change in wavelength. From the experimental measurements of the gravitational red shift, and the Pound–Rebka experiment [7], the wavelength shift for a photon in a gravitational field is:

\[ \lambda = \lambda_0 \left(1 - \frac{\mu}{r}\right)^{-1} \] (4)

And thus the vertical index of refraction for light is:

\[ \eta_v = \left(1 - \frac{\mu}{r}\right)^{-1} \] (5)

Combining Eq.(1), and Eq.(5), gives the flat Minkowski space, anisotropic index of refraction. the same as Eq.(1.3), (see Appendix I.)

The Schwarzschild radius of the gravitational potential in GR results from energy transfers to the field which creates an event horizon. For a local energy conservation theory, the barrier at the Schwarzschild radius does not exist and thus the expression must be the quadratic alternative:

\[ \left(1 - 2\frac{\mu}{r}\right) \rightarrow \left(1 - \frac{\mu}{r}\right)^2, \] (1.6)

For such a theory the Karimi expression Eq.(1.2), and the heuristically derived expression Eq.(2.4), must also be modified to the quadratic version:
\[ c = \left( 1 - \frac{\mu}{r} \right)^{\left( 1 + \cos^2 \theta \right)} \]  

(1.7)

The change from Eq.(2.4), to Eq.(1.7), has little effect on physical phenomena accept in the local proximity of a Black Hole.

**Conclusion**

Expression Eq.(1.7), is the proper expression for the asymmetric propagation of light in a theory of gravitation in which there is local conservation of energy.

References:


4. DT Froedge, Gravitation is a Gradient in the Velocity of Light V081216, http://www.arxdtf.org/


Appendix I

Expression (1.7) is arrived at by noting that:

\[
\eta_r = \left(1 - \frac{\mu}{r}\right)^{-1}
\]

(2.1)

\[
\eta_\phi = \left(1 - 2\frac{\mu}{r}\right)^{-1}
\]

(2.2)

The coordinate velocities are then:

\[
c_x^2 + c_y^2 = \frac{v^2 \cos^2 \theta}{\left(1 - \frac{\mu}{r}\right)^4} + \frac{v^2 \sin^2 \theta}{\left(1 - \frac{\mu}{r}\right)^2}
\]

(2.3)

Rearrangement and dropping third order terms gives:

\[
c = \left(1 - \left(1 + \cos^2 \psi\right)\frac{\mu}{r}\right)
\]

(2.4)

Appendix I

Shapiro Delay and Deflection

For a solar grazing photon, it can be shown that Eq.(1), gives the proper Shapiro time delay, and thus by Fermat’s principle gives the proper deflection
For a photon having $R$ as the closest approach to a gravitation body and Eq.(1.3), as the index of refraction, the change in the distance traveled as the result of the gravitating body is

$$
\Delta vt = -\mu \int \left(1 + \cos^2 \theta \right) \frac{dt}{r} \tag{3.1}
$$

From fig. 1

$$
\cos \theta = \frac{R}{r} \tag{3.2}
$$

Thus:

$$
\Delta vt = -\mu \int \left(1 + \frac{R^2}{r^2} \right) \frac{dt}{r} \tag{3.3}
$$

Or:

$$
\Delta vt = -\mu \int \left(\frac{1}{r} dt + \frac{R^2}{r^3} dt \right) \tag{3.4}
$$

In units of $c=1$ the $r$ is:

$$
r = \sqrt{R^2 + t^2} \tag{3.5}
$$

$$
\Delta vt = -2\mu \int_0^t \left(\frac{1}{\sqrt{R^2 + t^2}} dt + \frac{R^2}{(R^2 + t^2)^{3/2}} dt \right) \tag{3.6}
$$

This is the exact expression for the Shapiro delay arrived for GR by[8] Sec10.2, thus the index of refraction of light shown in Eq.(1), gives the correct delay consistent with the GR predicted and measured values.
By virtue of the fact that by Fermat’s principle or the principle of least action the time delay defines the angular deflection, thus the angular deflection $2\mu / R$ caused by the delay, is the same as that of GR.